

Homework 2

Due: 18 Sep 2023

REMEMBER:

- Include work in symbolic form (e.g. $p(X = \text{foo} | Y = 3)$). I should be able to tell where every number came from. You can abbreviate (e.g. $p(\text{foo}|3)$) as long as it's clear what is meant.
- Sanity-check your answers. If your answer seems crazy but you can't find the error, at least make it clear you know there's a problem.

Problem 2.1

A standard deck of playing cards has 52 cards in four suits (two red, two black), each suit with cards numbered 2–10 and “face cards” labelled jack, queen, king, and ace. A “pinochle deck” has 48 cards: each of the four suits has only 9 and 10 plus the face cards, and each card appears twice in the deck.¹

Assuming in each case that the relevant deck is well-shuffled, identify the following probabilities. Show your work by making it clear where each number in your probability comes from.

- a. $p(R=\text{ace} \mid D=\text{standard})$: In a standard deck, the probability of drawing any card whose rank is ‘ace’
- b. $p(R=\text{ace} \mid D=\text{pinochle})$: In a pinochle deck, the probability of drawing any card whose rank is ‘ace’
- c. $p(F=\text{true}, C=\text{red} \mid D=\text{pinochle})$: the probability of drawing a card whose rank is one of the face cards and whose suit is one of the red suits, from a pinochle deck
- d. $p(R=A, S=H \mid F=\text{true})$: the probability of drawing the ace of hearts given that the card drawn is a face card
- e. If there are 3 standard decks and 1 pinochle deck in a pile, and you pick one at random before drawing a card from that deck, what is $p(F=\text{true})$, i.e. the overall probability of drawing a face card?

¹I swear I am not making this up. It's pronounced “PEE-nuckle”.

Problem 2.2

I have many decks of playing cards, most of which are standard, but some are pinochle decks—for the purposes of this problem, let's say I have two pinochle decks and ten standard decks. Without counting the cards, it's hard to tell at a glance whether you've accidentally grabbed a pinochle deck.

- a. If I grab a deck completely at random and draw a card from it, what is the probability that the card is a 5?
- b. If I grab a deck completely at random and draw a card from it, what is the probability that the card is a jack?
- c. If I grab a deck completely at random and draw a card, and the card is a jack, what is the likelihood that I've grabbed a pinochle deck?
- d. If I grab a deck completely at random and draw two cards from it, some pairs give me certainty: if either card is a 4, for instance, or if both cards are the jack of diamonds. But if one is a jack and the other is a king, does that give me any knowledge about the deck? Why or why not? (Note: I'm not looking for exact numbers on this part, because they're subtle and a bit gross. Focus on the analysis.)

Again, don't forget to show your work.

Collaboration policy: group work! If you work with other people on this homework, hand in one copy and put all your names on top. There will be a revision cycle for this.