

Homework 3

Due: 11 February 2025

As with previous homeworks: the theoretical part is in L^AT_EX and group work and will have a revision cycle; the practical part is programmed and collaborative-but-not-group work and won't have a revision.

Problem 3.1 — theoretical

Prove that for all natural numbers n , $2^n \geq 1 + n$. Use induction.

(You may use any mix of two-column and paragraph proof here, but I encourage a two-column for at least the more involved parts.)

Problem 3.2 — practical

Write the following Racket functions and their test cases, using recursion:

- a. A function `countdown` that, given a nonnegative integer n , builds a string that counts down to a final "BLASTOFF!"

(For instance, `(countdown 3)` should return "3 2 1 BLASTOFF!".)

Note that here the "recursive case" is $n - 1$, similar to how we set up a proof that used numeric induction.

- b. A function `concat-with-spaces` to concatenate all the strings in a given list of strings (but starting and ending and separated with spaces, exactly one space at a time, i.e. no double spaces).

(For instance, `(concat-with-spaces (list "foo" "bar" "baz"))` should return " foo bar baz ".)

Problem 3.3 — practical

Write the following Racket functions and their test cases:

- a. A function `any-positive-integers?` that determines whether any of the values in a given list are positive integers.
(Note that Racket has `positive?` and `integer?` the former of which requires that its given value is at least a number.)
- b. A function `only-positive-integers?` that determines whether every value in the given list is a positive integer.
- c. EXTRA: A function `filter-positive-integers` that builds a list containing all and only the positive integers in the given list. (Hint: your combining function here will be `cons` itself!)

Hand in the file(s) containing the proof and the Racket functions using the `handin` script:

```
handin cmsc208 hwk3 proofs.tex myfile.rkt [myfile2.rkt ...]
```

Collaboration policy, as stated at the beginning: **For Problem 3.1:** group work! If you work with other people on this homework, you can just hand in one copy and put all your names on top. There will be a revision cycle for this. **For Problems 3.2–3.3:** collaborative. You each have to hand in your own version of the assignment, but you can talk to other people about the problems. Mention in a comment or readme who you worked with. (Still no copying, though.)