Homework 3

Due: 6 February 2024

As with previous homeworks: the theoretical part is in LaTeX and group work and will have a revision cycle; the practical part is programmed and collaborative-but-not-group work and won't have a revision.

Problem 3.1   — theoretical

Prove that for all natural numbers n, $2^n \geq 1 + n$. Use induction.

(You may use any mix of two-column and paragraph proof here, but I encourage a two-column for at least the more involved parts.)

Problem 3.2   — practical

Write the following Racket functions and their test cases:

a. A function `countdown` that, given a positive integer n, builds a string that counts down to a final `"BLASTOFF!"`

   (For instance, (`countdown 3`) should return `"3 2 1 BLASTOFF!"`.)

   Note that here the "recursive case" is $n - 1$, similar to how we set up a proof that used numeric induction.

b. A function `concat-with-spaces` to concatenate all the strings in a given list of strings (but starting and ending and separated with spaces).

   (For instance, (`concat-with-spaces (list "foo" "bar" "baz")`) should return `" foo bar baz "`.)

Problem 3.3   — practical

Write the following Racket functions and their test cases:

a. A function `any-positive-integers?` that determines whether any of the values in a given list are positive integers.

   (Note that Racket has `positive?` and `integer?` the former of which requires that its given value is at least a number.)

b. A function `only-positive-integers?` that determines whether every value in the given list is a positive integer.

c. EXTRA: A function `filter-positive-integers` that builds a list containing all and only the positive integers in the given list. (Hint: your combining function here will be `cons` itself!)

Hand in the file(s) containing the proof and the Racket functions using the handin script:

```
handin cmsc208 hwk3 proofs.tex myfile.rkt [myfile2.rkt ...]
```

Collaboration policy, as stated at the beginning: **For Problem 3.1:** group work! If you work with other people on this homework, you can just hand in one copy and put all your names on top. There will be a revision cycle for this. **For Problems 3.2–3.3:** collaborative. You each have to hand in your own version of the assignment, but you can talk to other people about the problems. Mention in a comment who you worked with. (Still no copying, though.)