Binominal *each*: evidence for a modified type system

by
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The word *each*, when attached to a noun phrase as in the sentence "The kids bought three books each.", has a number of unusual syntactic and semantic properties. It operates on two different noun phrases in the sentence; the Dist phrase, to which *each* attaches, must be numeric and indefinite, while the other noun phrase—the Range—may be definite or indefinite so long as it is plural. The Relation between those two noun phrases need not be a transitive verb; nearly any two-place relation will do. In this thesis I analyse this situation and develop a theory that accounts for these observations. In so doing, I use this "binominal *each*" construction to argue for the basic correctness of certain pre-existing linguistic theories, and for a few specific modifications thereto.

Vita

Don Blaheta was born on 28 March 1978 in Chicago, and continued to reside in Illinois through grade school, high school, and his undergraduate years at Quincy University (in Quincy, Illinois), where he graduated *cum laude* with bachelor's degrees in mathematics and in computer science. He moved to Providence in 1997 to join the PhD program in Computer Science at Brown, and achieved an ScM degree in that department in 1999; he plans to defend his dissertation in that department this summer and officially receive his PhD next May. This Fall he will be returning to Illinois, where he has been appointed to the position of Assistant Professor of Computer Science at Knox College, in Galesburg.

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This is an excellent place to thank the National Science Foundation for funding me, through the Integrative Graduate Education and Research Traineeship (IGERT) program—a great deal that enabled me to broaden my horizons, and gave me the time to pursue this master's degree outside my primary field.

And now, before the music comes up and the microphone descends into the floor, I'd like to thank the members of the Brown Ballroom Dance Team, the Brown Renaissance Singers, the Brown-RISD Catholic Community, and the `imsasun` regulars, as well as the whole Coconut Lounge crowd, for their assistance in sanity maintenance. Special thanks go to Matt Sherman, for taking time off from his own thesis for some excellent and much-needed late-night conversation; to Theresa Ross, for great conversations, a fabulous roadtrip, and my newfound obsession with knitting; to Kathleen Corriveau, for many years of fun dancing and good times; to Sharon Goldwater, for sharing my interests in computer science, linguistics, *and* dancing, and for her insight into and perspective on all of those things; and to Mike Kimmitt, for his contagious political fervour, for his welcoming spirit, and most for his dozen-years-strong friendship.

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Chapter 1

Introduction

Several years ago, I read Fred Landman's article simply entitled "Plurality" [6], which enumerates the various different available distributive and collective readings available for most verbs, and developed an explanation for how to generate these readings. Many example sentences were given, most of which had at least two and some as many (Landman claimed) as eight; the default reading varied from sentence to sentence. One sentence caught my eye:

Four hundred fire fighters put out twenty fires. (1.1)

For mostly pragmatic reasons, this sentence strongly prefers a collective reading; nevertheless, I observed, the fully distributive meaning could be forced by appending the word *each* to the sentence, much as the collective reading could be confirmed by appending the word *altogether*.

This observation was followed immediately by a number of others, on how interesting these words were, to be selecting the distributivity of the verb phrase, not as modifiers on the VP itself, but on its direct object. Not necessarily even the direct object: *each* could be attached any number of different places, including either object of ditransitive constructions, PP complements, adjunct modifiers, and free noun phrases. It seemed like it would be tricky and interesting to integrate into a surface-compositional account of the semantics of English; and thus I decided to give it a try.

Safir and Stowell [10] were not the first to notice this construction, but they seem to have been the first to take a systematic look at the properties of (and give a name to) "binominal *each*". Their listing of exemplars of the construction is fairly comprehensive, and forms the basis for my own listing in Chapter 2.

Chapter 3 then provides a brief, broad overview of what I hope to accomplish in the end, and the tools I use to do so.

Next, in Chapter 4, I cover the main prior proposals on the binominal *each* construction: Safir and Stowell's, as previously mentioned, and especially Malte Zimmerman's. Zimmerman [15] gives a detailed syntactic and semantic analysis of *each* and its German analogue *jeweils*, along with a broad cross-linguistic analysis of similar "d-distributive" constructions.

Chapters 5 and 6 are the meat of the thesis; it is there that I develop my category and type system, and demonstrate how these support a fully surface-compositional analysis of many of the binominal *each* subtypes. The three main restrictions on the *each* construction—plurality, cardinality, and the clausemate restriction—will each be handled, the first as a stipulation but the other two as part of the system.

I conclude in Chapter 7 with speculation on some yet-unresolved issues, and finally a summary of my own contributions in this work.

The appendices contain related items that would have been cumbersome in the main body of the thesis. Appendix A explores the ramifications of my modified type system on some of the well-known properties of quantifiers. Appendix B gives full derivations for most of the denotations given in the text. Finally, Appendix C collects several of the definitions scattered through the text into one place for easy reference.

Chapter 2

The menagerie

In this chapter we will present a number of examples of where 'each' and related words can occur. While not meant to be *entirely* exhaustive, we do hope that most of the major cases are covered. A few of the examples are not handled by our theory, and they will be discussed further in Section 7.1.

Some of the observations made in this Chapter were first made elsewhere. In particular, most of the different possible cases for the Range and Dist phrases can be found in [10], while a few of the different Relations are seen in [15].

2.1 The basic case

The canonical form of the construction we are considering is a simple subject-verb-object sentence that ends in the word 'each':

The kids bought three books each. (2.1)

There are a number of semantically similar sentences involving the word *each*, which are only tangentially relevant to the issues discussed in this thesis. First of all, there is the *each* that is a determiner, combining with a bare (singular) noun to form a noun phrase:

Each kid bought three books. (2.2)

If there is a clear antecedent in a previous sentence, the noun can be omitted:

...the kids. Each bought three books. (2.3)

Next, there is the adverbial *each*, that modifies the verb phrase:

Alex and Sasha each bought three books. (2.4)

(This version is often called "floated", because in complex verb phrases it can float around, as in "Alex and Sasha each would have bought...", "...would each have bought...", "...would have each bought....")

While these instances are semantically related, this work is primarily concerned with the *each* shown in (2.1), called variously "binominal"[1] [10] or "adnominal" [15]. The former term is descriptive of how it is necessarily involved with two noun phrases—it distributes one noun phrase over the members of another. The latter term is significant of the fact that syntactically, it seems to attach to the noun phrase itself, rather than acting as an adverbial modifier: if it were adverbial, we would also expect a postposed *each* to work on intransitive verbs, which it clearly does not:

The kids each slept. (2.5)

*The kids slept each. (2.6)

2.1.1 Anatomy of the binominal *each* construction

Aside from the word *each* itself, there are three major components to the construction.

$$\underbrace{\text{The kids}}_{\text{Range}}\ \underbrace{\text{bought}}_{\text{Relation}}\ \underbrace{\text{three books}}_{\text{Dist}}\text{each.}$$

Figure 2.1: The canonical case, labelled

The Dist phrase is the noun phrase immediately adjacent to the word *each*. It is canonically the object of the sentence, "three books" in Figure 2.1. Safir and Stowell call this the DistNP; Zimmerman the DistShare. It is the thing being distributed across multiple people. For convenience, we will sometimes refer to the Dist phrase together with the attached *each* as the *each*-phrase.

The Range phrase is the noun phrase further away from *each*. It often acts as the subject of the sentence, "the kids" in Figure 2.1. Safir and Stowell also call it the RangeNP, but Zimmerman names it the DistKey. It is the group being distributed over.

Finally, the Relation is typically a verb, "bought" in Figure 2.1. It represents the relationship between individual members of the denotation of the Range phrase and anonymous instances of the Dist phrase.

2.2 Valid Range phrases

The main restriction on the Range phrase is that it be plural. It can be a definite phrase:

Alex and Sasha bought three books each. (2.7)

Those kids bought three books each. (2.8)

Alex and the other kid(s) bought three books each. (2.9)

or an indefinite phrase:

Some kids bought three books each. (2.10)

Two kids bought three books each. (2.11)

[1] Often accidentally cited as "binomial".

It cannot be singular, whether definite or indefinite:

*One kid bought three books each. (2.12)

*A kid bought three books each. (2.13)

*Alex bought three books each. (2.14)

In some conditions, even generic plurals can be acceptable, as in the following example from [10]:

Martian men marry two women each. (2.15)

Examples involving generalised quantifiers vary somewhat in their acceptability, but it seems that those quantifiers that are most distributive in nature are most bad.

Some kids bought three books each. (2.16)

?All the kids bought three books each. (2.17)

*Every kid bought three books each. (2.18)

*Each kid bought three books each. (2.19)

The type of plurality required is somewhat open for discussion. Morphological plurality is not sufficient:

*The trousers have three buttons each. (2.20)

This sentence fails on the reading with a single pair of trousers. On the other hand, semantic plurality doesn't necessarily guarantee anything either:

?The committee read three papers each. (2.21)

?The group carried two bags each. (2.22)

Some people report sentences (2.21) and (2.22) to be perfectly valid, while others have some level of difficulty with them (often indicated as rephrasing before confirming validity: "The members of the committee…"). This judgement does not run along the same lines as the American English vs. Commonwealth English distinction between "the committee is" and "the committee are"—at least some informants prefer the former but find the sentences valid.

2.3 Valid Dist phrases

The Dist phrase must be indefinite and cardinal—meaning it has a number or something very like a number modifying it. It need not, however, be plural.

Alex and Sasha bought one book each. (2.23)

Alex and Sasha bought several books each. (2.24)

Definite phrases are definitely bad:

*Alex and Sasha bought the book(s) each. (2.25)

*Alex and Sasha bought those books each. (2.26)

Interestingly—and importantly—indefiniteness does not suffice. If there is not a number associated with the Dist phrase, it is unacceptable.

*Alex and Sasha bought some books each. (2.27)

*Alex and Sasha bought books each. (2.28)

The word *a* occupies a fuzzy middle ground between being a number and being a determiner. Its acceptability in a Dist phrase varies from person to person, and indeed varies according to context:

?Alex and Sasha bought a book each. (2.29)

Alex and Sasha paid a dollar each. (2.30)

Note that the Dist phrase restrictions do not at all apply to the equivalent sentences that use floated *each*:

Alex and Sasha each bought the book(s). (2.31)

Alex and Sasha each bought those books. (2.32)

Alex and Sasha each bought some books. (2.33)

Alex and Sasha each bought books. (2.34)

Alex and Sasha each bought a book. (2.35)

The cardinality restriction is interesting; the numbers can get quite complex, and the main limitation seems to be that the sentence eventually becomes unwieldy:

Alex and Sasha bought a couple books each. (2.36)

Alex and Sasha bought a few books each. (2.37)

Alex and Sasha bought at least three books each. (2.38)

Alex and Sasha bought at most three books each. (2.39)

Alex and Sasha bought between three and five books each. (2.40)

Alex and Sasha bought at least three but certainly no more than five books each. (2.41)

2.4 Other constructions using binominal *each*

The Relation between the Range and Dist phrases can likewise become relatively complex. On ditransitive verbs such as *give*, the Range phrase can be either the first or second argument, and the

Dist phrase either the second or third.

Alex and Loren read two kids each a story. (2.42)

Alex read Sasha and Chris three story each. (2.43)

Alex and Loren read Sasha three story each. (2.44)

This is also true of ditransitives that have undergone dative shift, and of prepositional ditransitives as well.

Alex and Loren read three stories each to Sasha. (2.45)

Alex read Hamlet, Othello, and King Lear to two kids each. (2.46)

Alex and Loren read Hamlet to two kids each. (2.47)

In fact, if the Dist phrase is the third argument of a ditransitive verb and both of the other two are eligible to be a Range phrase, there is a preference for the second but an ambiguity exists between the two interpretations. This is discussed in greater detail in Section 6.2.

The verb phrase can undergo all the usual VP tense and mood modifications, participate in control structures, etc.:

The kids did buy three books each. (2.48)

The kids needed to buy three books each. (2.49)

The teacher told the kids to buy three books each. (2.50)

The kids' minds were occupied with needing to buy three books each. (2.51)

The Dist phrase can also be the object of a preposition. It can be part of the verb phrase, as complement, or attached to the verb phrase, as adjunct:

Alex and Sasha relied on three assistants each. (2.52)

Alex and Sasha dragged three bags through four puddles each. (2.53)

Alex and Sasha hung the curtains over two windows each. (2.54)

Alex and Sasha staffed the stores for two nights each. (2.55)

The prepositional phrase can also be attached to a noun:

Alex and Sasha visited the capitals of three states each. (2.56)

Mickey wears gloves with four fingers each. (2.57)

Note that in (2.56), we have Alex and Sasha as the Range, with a Relation of "visited the capitals of", while in (2.57) the Range is the gloves—the noun being modified by the prepositional phrase—and the Relation is genitive, represented in the sentence just by the word *with*.

The Dist phrase can even be a free NP acting adverbially:

Alex and Sasha told three stories four times each. (2.58)

As with the ditransitives and several of the prepositional cases, the Range phrase can be ambiguous between the two earlier noun phrases.

2.4.1 Boundary cases

It is not clear how restricted the Range phrase is. Can it be *any* previous noun phrase? Not quite, but the restrictions may be somewhat more pragmatic than absolute.

The boys liked three books each. (2.59)

?The boys liked, and told Mary to read, three books each. (2.60)

?The boys told Mary to read three books each. (2.61)

*The boys believed that Mary read three books each. (2.62)

?The boys hated, and thought that Mary loved, three books each. (2.63)

The questionable examples here can be achieved to some extent on a *de re* reading—i.e. for each boy there are three specific books involved, though we may not know what they are.

There seem to be some (admittedly contrived) situations where changing seemingly irrelevant parts of the sentence change the acceptability of the construction—this may admittedly be pragmatic as well:

The profs on Bob's committee told him to read three books each. (2.64)

?The profs on his committee told Bill to read three books each. (2.65)

In parallel constructions and in questions (and their answers), the Dist phrase can be slightly different in composition or location than described above. It's possible that these are just examples of a word or phrase undergoing movement or being elided, and that the correct analysis works as described above in some sort of underlying structure. Nevertheless, we record them as possible boundary cases:

How many apples did the boys eat? Three each. (2.66)

How many books did the boys tell Sarah to read? Three each. (2.67)

How many books each did the boys tell Sarah to read? Three. (2.68)

Sarah ate (just) one apple but the boys ate three each. (2.69)

Finally, we have recorded some actual utterances that demonstrate some unusual uses of *each*:

We ran into five people we know, each. (2.70)

How many slices (of pizza) can we have each? (2.71)

2.5 Other binominals

There are other words that attach to noun phrases and act in a similar fashion to *each*. The word *apiece* seems to be most similar—it's definitely binominal and adnominal, and can substitute for *each* in most sentences:

Alex and Sasha bought three books apiece. (2.72)

The books cost ten dollars apiece. (2.73)

Loren gave the children four lollipops apiece. (2.74)

A difference is that it has a slight preference for distributing over animate objects:

Three kids recited four poems two times each/apiece. (2.75)

Here the *each* version seems to prefer the poems as its Range, but the author and several other native English speakers find that the *apiece* version prefers the kids as Range. It isn't just a preference for the subject; if the lower NP is a valid Range phrase and animate, it will be preferred:

The adults told the kids two stories apiece. (2.76)

The ambiguity is present, but again the lower argument is the preferred Range. Animacy is not required, as seen in example (2.73).

Altogether can occur adnominally in most of the same places as *each*, and means roughly the opposite.

Alex and Sasha bought three books altogether. (2.77)

The kids bought one book altogether. (2.78)

Chapter 3

A broad overview

For the remainder of this thesis, we will be attempting to explain the phenomena enumerated in the previous chapter.

First, we will recount the prior analyses of the *each* construction. Safir and Stowell's account [10] is primarily syntactic, but provides the baseline on which further work (including our own) is built. Zimmerman [15] provides a much more detailed system, including syntactic and semantic observations and a great deal of cross-linguistic data.

However, Zimmerman's account is (crucially) not surface-compositional. Hence our work, which began as an attempt to explain all the peculiar properties of the *each* construction in a compositional fashion, without recourse to transformations from an underlying form. To whatever extent possible, we hoped to explain it using nothing more than function application and composition, and (a minimum of) unary grammar rules.

We believe we have largely succeeded. In Chapter 5, we develop the details of our category and type systems; of which the major innovation will be a new category NumP that includes the sort of phrases that can attach to *each.* Also in this chapter, we will explicitly detail the theory of plurality that we will be using in our *each* analysis—that plurals are sets of entities, and singulars are singleton sets of entities—along with the changes this will cause in the traditional systems of verbs and quantifiers. (Appendix A will deal specifically with the question of how our modified type system handles the observations made by Barwise and Cooper [2] and Keenan and Stavi [5] regarding properties of generalised quantifiers—notably their conservativity.)

Having laid out a slightly modified type system, in Chapter 6 we work through a number of different configurations of the *each* construction, and develop a lexical denotation for the word *each* that will work in all of them, correctly predicting which variations are valid English sentences (and giving their meanings), as well as blocking those variations that are not valid. We give the denotations that our system derives for all of these, and the derivations themselves we have listed in Appendix B.

Finally, we will discuss some limitations on and unexplored areas of our framework, along with speculation on how these cases might be handled.

Chapter 4

Prior proposals

4.1 Safir and Stowell (1989)

The problem of binominal *each* was first discussed at length by Ken Safir and Tim Stowell [10], who gave a long list of examples and a primarily syntactic analysis. Only the subject-verb-object case was considered, although a variety of possible Range and Dist phrases are enumerated (many of which are reproduced in Chapter 2).

The essence of Safir and Stowell's analysis was that binominal *each* had the same structure as partitive *each*. The partitive construction has *each* in the Spec position of an NP; if the word *one* is present, it heads the $\overline{\mathrm{N}}$[1] and is modified by the *of* phrase that actually creates the partition. Safir and Stowell argue that even when the *one* is not overt, the structure is the same, and the head of the NP is present but covert, as shown in Figure 4.1 below.

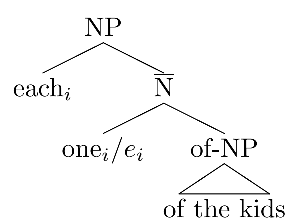

Figure 4.1: Syntactic analysis of partitive *each*

Binominal *each* can then be seen as having an identical form to the partitive construction, but with an also-covert of-NP; this can then be coreferent with the Range phrase. A Safir-and-Stowellian analysis of the canonical example (given earlier as (2.1)) is given in Figure 4.2 on the following page.

[1] Although most of the recent literature has been using apostrophes or primes when discussing X-bar theory, this seems to have begun due primarily to typographical limitations. The apostrophe/prime notation is both ambiguous and no longer necessary, as modern computers are more than capable of generating the original notation. Hence, when we refer to an N-bar, we will notate it as $\overline{\mathrm{N}}$. Surprise!

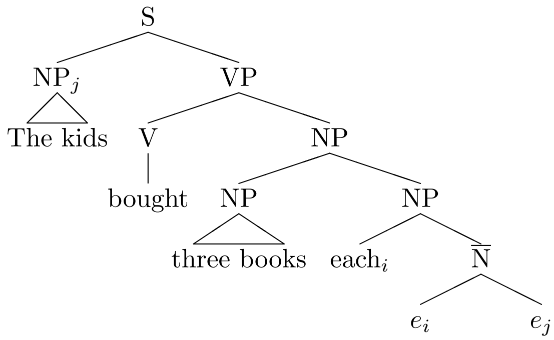

Figure 4.2: Safir and Stowell's analysis of (2.1)

Two main arguments are given in favour of this analysis. The first is that the right-heaviness of this NP explains why binominal *each* is postnominal, by analogy with adjectival phrases. That is, modifiers like "happy about his plans" or "willing to try" are placed after nouns, while the simpler "happy" and "willing" would normally be placed before the nouns. Similarly, "each *e e*" might be 'heavy' enough to cause it to be a postnominal modifier.

The second argument is that the anaphoric coreference between the covert of-NP and the Range phrase explains a number of restrictions on where various parts of the *each* construction can fall in a sentence. If we posit an anaphoric relation between *each* and the Range phrase, other syntactic rules governing anaphor can then be used to account for those restrictions.

The syntactic analysis of this paper is fairly thorough and, on the whole, convincing, but it does not give an account of the semantic mechanisms by which the *each* construction works. For this we turn to the dissertation of Malte Zimmerman.

4.2 Zimmerman (2002)

Malte Zimmerman [15] gives a much more general analysis of the problem, considering not just syntax but also semantics and pragmatics, and expanding the linguistic scope from primarily English to also highlight German and include a survey of distance-distributive constructions from around the world. The semantic analysis is somewhat compositional in the sense that the *each* construction does not undergo any movement, and thus can be interpreted *in situ*, mostly. Further more, the semantics can also influence the syntax: an otherwise syntactically valid construction, that can nevertheless not be interpreted, is therefore ruled out as a valid utterance.

Zimmerman does, however, treat the syntax and semantics separately, and we will present our summary in the same way.

4.2.1 Syntax

By considering distance-distributive constructions in many languages, Zimmermann tries to come up with a language-independent framework to put them all in. Specifically, he argues that *each* and its analogues are all composed of

- a prepositional item,
- a determiner, and
- a (restricted) noun phrase.

Any of these three elements can be covert in a given language, but at least one of the preposition and determiner must be overt. In English, we have only the determiner overt: *each*. Other languages have somewhat different structures; a few are presented in Table 4.1. Most combinations seem to be attested in some language.

Language	***each* word/phrase**	**Analysis**
English	each	P^0 each-e (cov. prep, det, cov. NP)
French	chacun(e)	P^0 chac-un(e) (cov. prep, det, NP)
German	jeweils	je-weil-s (det, NP, genitive-marking-as-prep)
Czech	po	po Q^0-e (prep, cov. det, cov. NP)
Russian	po kazhdyi	po kazhdyi-e (prep, det, cov. NP)

Table 4.1: Analogues of *each* in other languages

This structure just covers the actual word *each* or its analogue. As a prepositional phrase, then, it modifies its sister—the Dist phrase, an NP[2]—to form a new NP. This will then generally combine with a covert determiner to make the *each*-phrase, a DP. Syntactically, there seems to be no further restriction on where the *each*-phrase can occur; as a DP it should be able to occur in the usual variety of places. We know that it can only occur in certain parts of the sentence, in order to allow it to have a Range phrase, but this restriction is seen as semantic.

4.2.2 Basic semantics

The general framework within which Zimmermann is working is Davidsonian, an event-based system. Rather than having transitive verbs take two entities and return a truth value, this system instead has the return value be a set of events. This set can be intersected with modifiers like "in Rome" (the set of all events that took place in Rome) or "during the Ides of March" (all events taking place in mid-March). While the events themselves don't play a direct role in the semantics presented for *jeweils* (German) and *each*, they do inform the theory, and the Davidsonian influence will be seen in how the meaning of *each* composes with the rest of the sentence.

First, we will discuss the simple lexical meaning of the word *each* (or rather, of the prepositional phrase "each", according to Zimmerman's theory). We might verbalise the basic meaning as, "every member of group Z stood in relation R to some group of things P." This is simply

$$[\![\text{each}]\!] \quad = \quad \lambda P.\forall z[(z \in Z_i) \rightarrow \exists x[P(x) \wedge R_j(z,x)]] \tag{4.1}$$

[2]Note that Zimmerman subscribes to the syntactic theory (generally attributed to Steve Abney [1]) that recasts the traditional noun phrase as a "determiner phrase", or DP; as such, the theory's NP is what might elsewhere be called $\overline{\text{N}}$.

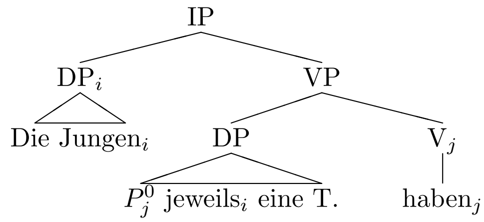

Figure 4.3: Analysis of a German sentence using *jeweils*

Note that in this definition, Z and R are free. Z coindexes with the Range phrase, and R with the Relation. P, the only variable bound by a lambda, is the Dist phrase. Presumably this is safe because the object of *each* is always adjacent to it, while the others are left free because there can be intervening bits of the sentence preventing a straight-up vanilla function application.

The *each*-phrase composed with its object make up a DP, which typically is the object of the sentence. To get the entire meaning of the whole construction (including Range and Relation), we now need to further compose this DP with the rest of the sentence. Because the definition includes the meaning of the Range and Relation as free variables Z and R, the type of this DP ("two books each") is actually t.

This is where the Davidsonian influence alluded to above can be seen. Rather than trying to compose the sentence meaning primarily through function application, Zimmerman takes the fundamental mode of semantic composition as modification, introducing rules on how to combine two elements, according to properties like type and available free variables.

Here, in the basic *each* case, the rule introduced is called *index-triggered lambda abstraction*; this says that if you have two siblings to combine, then if one has an index and the other has a free variable with that index, you can *then* do the lambda abstraction that will let you substitute the one for the other. Further discussion on the indexation mechanism is given after the example below.

The *each*-phrase need not occur as the direct object of a transitive verb, of course. It can occur as the indirect object or as an adjunct; or even as the object of a preposition modifying an NP. In all cases, though, it requires a relation-denoting expression, be it transitive verb, preposition, or whatever. This provides the value for R in the meaning of the PP (coindexed with the P^0). See Chapter 2 for examples.

Furthermore, while in the most basic case the Range phrase is a simple plurality (e.g. "Alex and Sasha", "the children"), in some cases it will be a predicate (e.g. bare plurals, like "gloves"). To handle this case, Zimmerman makes use of another mechanism: after the index-triggered lambda abstraction has occurred, rather than using function application he uses predicate modification (i.e. intersection); this will be discussed in greater detail in Section 4.2.4.

An example

Figure 4.3 on the preceding page diagrams an example given by Zimmerman, repeated below as (4.2.2) with glosses and translation:

"...weil	die	Jungen$_i$	jeweils$_{i,j}$	eine	Tätowierung	haben$_j$."
because	the	boys	each	one	tattoo	have

"...because the boys$_i$ have$_j$ one tattoo each$_{i,j}$." (4.2)

The meaning ascribed to the *each*-phrase ("jeweils eine Tätowierung") is

$$\forall z[(z \in Z_i) \rightarrow \exists x[1\text{tattoo}'(x) \wedge R_j(z,x)]] \tag{4.3}$$

which is of type t, with two free variables. When this combines with the verb, there is a type mismatch, but the V is coindexed with a variable of the same type (R) in the meaning of the DP, which licenses lambda abstraction over that variable. Thus the value of the V is bound to R in the meaning of the DP in order to make the meaning of the VP:

$$\forall z[(z \in Z_i) \rightarrow \exists x[1\text{tattoo}'(x) \wedge \text{have}'(z,x)]] \tag{4.4}$$

This also is of type t, causing a type mismatch when combining with the subject, which is resolved in the same way: the subject DP ("die Jungen") is coindexed with the Z variable, so we lambda abstract. The meaning of the subject DP is bound to Z, yielding

$$\forall z[(z \in [\![\text{the boys}]\!]) \rightarrow \exists x[1\text{tattoo}'(x) \wedge \text{have}'(z,x)]] \tag{4.5}$$

The coindexation is the main binding mechanism for this process; that is, the meanings of the relation and the Range are made part of the meaning of the whole 'each' construction by means of being coindexed with the R and Z variables, respectively. It appears that there is no *a priori* index assignment,[3] but we might imagine a principle of charity here—indices are assigned in order to give a sensical reading of the phrase. Different index assignments are not explicitly disallowed, but they would simply not make sense.

A short digression on types

Zimmerman believes that the type of a plurality is simply a set of individual entities, or $\langle e,t\rangle$. However, he also argues that Linkian 'plural individuals' of type e would work as well for this framework. Since the choice is roughly orthogonal to the current discussion, we will use E for the type of pluralities; this has the advantage of also rendering some of the more complex types more readable.

4.2.3 Restrictions

Section IV.5 of Zimmerman's book deals with the properties of binominal 'each' that restrict where it can occur.

[3] On p 224, just before (157) and (158), Zimmerman says "The structure of the embedding DP, *including suitable indices*, is given in (158)." The use of the word 'suitable' here would seem to indicate that index assignment is not part of the theory.

The indefiniteness restriction

The Dist phrase needs to be indefinite. We cannot, for instance, say

*Die	Jungen$_i$	lieben	jeweils$_i$	diese	Frau.	(4.6)
The	boys	love	each	this	woman	

*The boys love this woman each.

or anything else using a definite NP (e.g. involving proper names, or *the*) for the Dist phrase.

In Zimmerman's theory, this falls out naturally from the exact meaning of *each*: the type of the argument to *each* is $\langle e, t \rangle$, which is the type of indefinite and numerical noun phrases in Zimmerman's framework. (Definite NPs are of type e, of course.) The type mismatch prevents any definite NP from appearing at that point in the sentence.

A further restriction that is not addressed by Zimmerman—though it was noted by Safir and Stowell [10]—is that the Dist phrase need be not only indefinite but actually numeric: the sentence "*The boys love some women each." is about as bad as the example above. Zimmerman gives a counterexample that would allow definite DPs as the Dist phrase, but it seems questionable in German and certainly does not work in English.

The plurality restriction

The Range phrase needs to be plural. Intuitively, this makes sense as the *each*-phrase needs more than one thing to distribute over. In Zimmerman's system, plural entities are of a distinct type from singular entities. As such, the restriction is again implemented through the type system: Z, a variable over pluralities, cannot be coindexed with an individual.

This slightly contradicts what he says elsewhere, namely that the type chosen for pluralities is orthogonal to the discussion. However, even if both individuals and pluralities are of type e, only pluralities can submit to the "x is a member of plurality y" operator that is necessarily part of the definition of *each*. Thus, the restriction still falls out naturally from the definition of *each*.

The clausemate restriction

The final restriction is in many ways the trickiest: the Range phrase and the Dist phrase must occur in the same clause. Consider the sentence

*Die	Verkäufer$_i$	sagen,	daß	Peter	jeweils$_i$	einen	Ballon	gekauft	hat.	(4.7)
The	merchants	say	that	Peter	each	one	balloon	bought	had	

*The merchants$_i$ say that Peter had bought one balloon each$_i$.

To get the reading indicated by the indices, the *jeweils* would need to cross the clause boundary indicated by *daß*; and this reading is definitely bad, both in German and in English. The theory's explanation for this is that the meaning of the verb combined with the each-phrase is a proposition (of type t) with a free variable Z (of type E) that represents the Range phrase. If this element

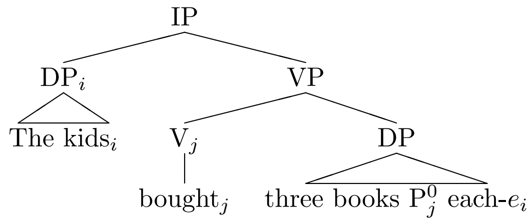

Figure 4.4: Zimmerman's ananlysis of (2.1)

does not immediately combine with the Range phrase, making use of the index-triggered lambda abstraction, then there is no way for it to combine.

In the specific case of this example, the VP "jeweils . . . hat" ("had . . . each") denotes a proposition (of type t) with a free variable. "Peter", its syntactic sibling, cannot be coindexed with Z due to being singular,[4] and there is no other rule in the grammar that will let us combine an e with a t. Consider also

Die	Verkäufer$_i$	sagen,	daß	die	Jungen$_j$	jeweils$_{j/*i}$	einen	Ballon	gekauft	haben. (4.8)
The	merchants	say	that	the	boys	each	one	balloon	bought	had

The merchants$_i$ say that the boys$_j$ had bought one balloon each$_{j/*i}$.

Here, the DP "die Jungen" *can* be coindexed, as it is plural; and it *must* be the argument of the lambda-abstraction because there is no other way for it to combine (again, we have no rule, other than the index-triggered abstraction, that lets an E combine with a t). This blocks the reading that distributes over the merchants ("die Verkäufer"), since the relevant variable is no longer free when that composition occurs.

4.2.4 Case analysis

I'll now give a number of examples, keeping as close as possible to Zimmerman's framework. In some cases these go into (considerably) more detail than the analyses presented by Zimmerman himself, so it's possible these do not exactly match what he would write.

Canonical form

On the canonical example, in (2.1), we give a Zimmermanian grammatical analysis in Figure 4.4; it yields the following denotation:

$$\begin{aligned} &[\![\text{The kids read three books each.}]\!] \\ &\quad = \forall z[(z \in [\![\text{the kids}]\!]) \to \exists X[\text{3books}'(X) \wedge \text{bought}'(z, X)]] \end{aligned} \tag{4.9}$$

This end meaning seems reasonable, and is (predictably) achieved through entirely straightforward application of Zimmerman's principles. In particular, index-triggered λ-abstraction occurs, first

[4] We say that it 'cannot' coindex with Z because it is singular and Z is plural; it might be more accurate to say that it could, but that the result would not make sense, and thus the coindexation is blocked.

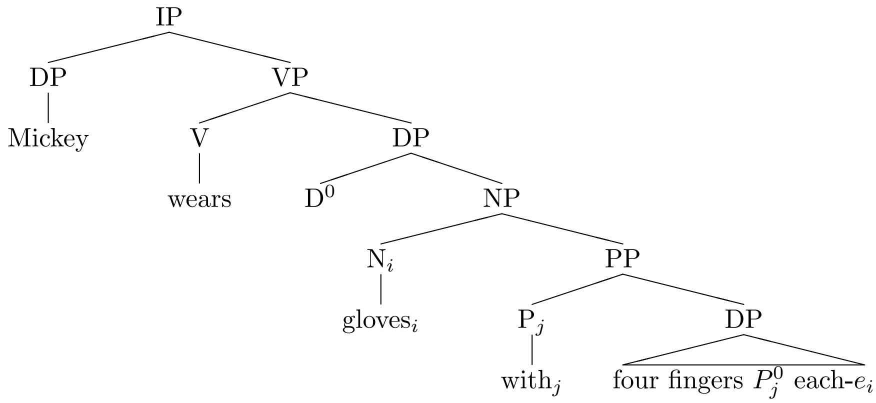

Figure 4.5: Zimmerman's analysis of a noun-modifying PP with *each*

when the *each*-phrase combines with the verb, then when the verb phrase combines with the subject. In the latter case, the VP denotation is a *proposition* of type t, with a free variable Z_i of type E:

$$\forall z[(z \in Z_i) \rightarrow \exists X[\text{3books}'(X) \wedge \text{bought}'(z, X)]]$$

When it tries to combine with the subject, a plural DP (of type E) which can be co-indexed with Z_i, a lambda abstraction occurs to bind the Z variable:

$$\lambda Z[\forall z[(z \in Z_i) \rightarrow \exists X[\text{3books}'(X) \wedge \text{bought}'(z, X)]]]$$

The type is now Et, capable of taking the subject as its argument; it does so and yields the denotation given above in (4.9). The full derivation is given in Appendix B.

Prepositional Relations

In example (2.57), we gave an instance of an *each*-distribution occuring entirely within a noun phrase, with the preposition acting as the Relation. Zimmerman's syntactic analysis of this construction is shown in Figure 4.5; for the noun phrase the denoation would be

$$\begin{aligned} &[\![\text{gloves with four fingers each}]\!] \\ &\quad = \lambda Z[\text{gloves}'(Z) \wedge \forall z[(z \in Z) \rightarrow \exists X[\text{4fingers}'(X) \wedge \text{with}'(z, X)]] \end{aligned} \tag{4.10}$$

The derivation proceeds largely as before, until the very end. The denotation for the PP is (again) a proposition, but the denotation for *gloves* is a predicate, not an entity; even with a λ-abstraction on the PP, the two denotations cannot compose through functional application. Nevertheless, the indices trigger a λ-abstraction

$$\lambda Z[\forall z[(z \in Z_i) \rightarrow \exists X[\text{4fingers}'(X) \wedge \text{with}'(z, X)]]]$$

which combines with the noun denotation

$$\lambda X[\text{gloves}'(X)]$$

through *intersection* (or "predicate modification"). The two predicates are of the same type, so they can combine to form a new predicate true only of those (plural) entities for which both original predicates were true. This results in the final denotation above in (4.10).

Indirect objects

Zimmerman's analysis of indirect objects is somewhat lacking. The sole case he considers is the one where *each* is on the indirect object itself; but in this case the direct object combines with the verb first, producing a two-place relation for the *each*-phrase to combine with. Here we repeat example (2.42):

Alex and Loren read two kids each a story. (4.11)

Zimmerman's analysis first builds the constituent "read _ a story" (in German this is contiguous), pronounces it to be equivalent to a transitive verb, and runs a derivation on that basis.

It seems clear that a true ditransitive analysis under Zimmerman's theory is possible, but either a new denotation is required for *each* that allows for three-place relations (and derives the resulting ambiguity), or rules will need to be devised for deriving the three-place denotations from the two-place denotation.

Chapter 5

A type system for pluralities

One of the main areas we have developed is a consistent theory of plurality that would work with a semantics for binominal *each*. Zimmerman's work neglects to discuss to discuss plurality in any great detail, except to claim that the exact theory of plurality did not particularly affect the rest of the theory. However, Zimmerman never really indicate what prevents singular Range phrases, how individuals and pluralities combine with the verb denotation, what exactly the verb denotation might be, or—most interestingly and importantly—how exactly to account for the cardinality restriction on the Dist phrase.

We will discuss our solution to these problems in Chapter 6, but first we will develop our theory of plurality and show its consistency on more basic examples.

5.1 Notation

As our examples become more complex, the most formal notation will become extremely unwieldy; thus, we have adopted a few conventions for notational convenience. Most are relatively common, but we enumerate them here for completeness.

Entities Represented by lowercase italic letters (a, h)

Sets For sets of entities—i.e. things of type $\langle e,t\rangle$—when it is otherwise clear I will sometimes write them without brackets, since the entities themselves are single letters. For instance, ac might be written as $\{a, c\}$. This is convenient when going one level up (to $\langle\langle e,t\rangle,t\rangle$): the usual $\{\{a, b\}, \{d, j, s\}\}$ is represented by the much more compact $\{ab, djs\}$.

Types Angle brackets tend to clutter, so types will in general be written without them. Since with rare exception removed brackets can be unambiguously recovered,[1] we will generally write et for $\langle e,t\rangle$, and $etett$ for $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$; brackets will sometimes be retained in a limited fashion to enhance legibility.

[1]Specifically, any e is grouped in a left-branching fashion with the sequence of ts that follow it; the resulting groupings are then themselves grouped in a right-branching fashion.

Variables Depending on the type and the semantic use of a variable, its form will differ:

Role	Type(s)	Description	Example
Sg. entities	*e*	lowercase italic x-series	x
Pl. entities	*et*	uppercase roman x-series	X
Sg. predicates	*et*	uppercase italic p-series	P
Pl. predicates	*ett*	uppercase roman p-series	P
GQs	*ett* or *ettt*	script p-series	$\mathscr{P}$
Relations	*eet* or *etett*	script r-series	$\mathscr{R}$

Table 5.1: Type and variable naming conventions

In addition, whenever a variable is introduced its type will be indicated underneath. In some cases where it makes sense to do so, the syntactic role of the variable will be superscripted as well.

5.2 Previous work

5.2.1 Distributivity: Scha (1980)

Some predicates are true of a group iff true of its members; others are more complex. There are a few different ways to deal with this complexity. Remko Scha [11] argued that all predicates should be over sets, even for "obviously" distributive predicates like *walk*. Any distributivity effects would then be a property of the verb rather than of the semantic system. He points out that a sentence like "the kids walk" is not precisely equivalent to "every kid walks"; this fact falls naturally out of a system where the distributivity of a predicate is a meaning postulate—living entirely in the semantics—rather than a syntactic or syntax-driven mechanism.

This view is elegantly simple, and we will be making use of it. In fact, the case of binominal *each* seems to argue for it, as we will explain in Section 5.3.3.

5.2.2 Plurals: Schwarzschild (1996)

There are two basic ways of looking at pluralities: structured sets (of sets), or flat unions. Roger Schwarzschild [12] prefers unions, and gives many reasons why; among other things, the type system is much simpler in the union case (though he definitely demonstrates that either model is feasible). We implicitly assume union-based pluralities in this work, but the primary relevance of the distinction is in complex conjunctive cases, which we don't really deal with. As a result, its main influence is that we assume all plurals, no matter how complicated, will have the same type: $\langle e, t \rangle$.

Schwarzschild also judges a number of things to be in the pragmatic domain; crucially, a major element in his theories of cumulativity and distributivity come from his notion of "covers", which specify exactly how one group is distributed over another. Covers, in this theory, are free and selected by context. This inspired us to similarly leave this out of the compositional semantics, although our theory ends up splitting the work between the lexical semantics and the pragmatics.

5.2.3 Plurals: Winter (2002)

For a starting point in our development of a type and semantic framework incorporating plurality, we have a number of systems to choose from; we have chosen that of Yoad Winter [14] as being most similar in outlook. His system is basically compositional, and works from the surface structure. However, a given syntactic category can have multiple corresponding semantic types; and thus it is the responsibility of the semantics to convert one or both denotations in a function-argument pair in order to resolve type mismatches.

Plural noun phrases are given the type *et*: sets of entities. Predicates can be either "atom predicates" or "set predicates". The former is comprised of verbs like *sleep* and *walk*, true of a set if and only if it is true of each member of the set; hence the predicate denotation is taken to be of type *et*, and converted to *ett* by the grammar as necessary to take plural arguments. Set predicates, on the other hand, are those verbs like *met* and verb phrases like "lift a piano", that can be true of a group without necessarily or even possibly being true of individual members. These are considered to be natively *ett*. However, there can be morphologically singular words (e.g. *group*, *committee*) that can meaningfully serve as the subjects of these verbs ("the committee met"); and since a singular noun phrase like "the committee" has type *e* under Winter's system, the verb is converted to type *et* for the purpose of taking these sorts of arguments.

Winter also discusses how numbers can build NPs, though it is not his primary focus and he doesn't go into great detail. Without suggesting an internal syntax for the NP, he seems to indicate that phrases like "exactly three kids" or "at most five books" are generalised quantifiers, although he notes that in the simple case ("four presents") the phrase is basically predicative.

5.2.4 Type shifting: Hendriks (1993)

Herman Hendriks, in his PhD dissertation [3] lays out a syntactic-semantic framework within which a given phrase has a single syntactic category, but inhabits a whole family of semantic types. This enables the denotations to be flexible, varying according to certain rules; in particular, it permits values of type *e* to be raised into values of type *ett*, and for predicates and relations expecting arguments of either type to accept arguments of the other type.

We will be making use of his Argument Raising rule. If a given constituent α has one valid denotation α', then another valid denotation can be derived according to this rule:

$$\mathrm{AR}\left(\underset{\langle\vec{a},\langle\langle s,b\rangle,\langle\vec{c},d\rangle\rangle\rangle}{\alpha'}\right) = \lambda\underset{\vec{a}}{\vec{x}}\ \lambda\underset{\langle s,\langle\langle s,\langle\langle s,b\rangle,d\rangle\rangle,d\rangle\rangle}{w}\ \lambda\underset{\vec{c}}{\vec{y}}.\,{}^{\vee}w\left({}^{\wedge}\lambda\underset{\langle s,b\rangle}{z}.\,\alpha'(\vec{x})(z)(\vec{y})\right) \tag{5.1}$$

Likewise for the Argument Lowering rule:

$$\mathrm{AL}\left(\underset{\langle\vec{a},\langle\langle s,\langle\langle s,\langle\langle s,b\rangle,d\rangle\rangle,d\rangle\rangle,\langle\vec{c},d\rangle\rangle\rangle}{\alpha'}\right) = \lambda\underset{\vec{a}}{\vec{x}}\ \lambda\underset{\langle s,b\rangle}{z}\ \lambda\underset{\vec{c}}{\vec{y}}.\,\alpha'(\vec{x})\left({}^{\wedge}\lambda\underset{\langle s,\langle\langle s,b\rangle,d\rangle\rangle}{u}.\,{}^{\vee}u(z)\right)(\vec{y}) \tag{5.2}$$

There are two main notational features of these definitions that should be explained. The 'up' and 'down' operators $^{\wedge}$ and $^{\vee}$ are used to convert between intensional and extensional meanings; the type s refers to the time-and-world index used in the intensional meanings.

The other interesting notation here is used to denote a series of zero or more arguments expected by a predicate. An argument to a lambda of type $\vec{a}$ represents some arbitrary number of separate lambda bindings, of unspecified type.

$$\lambda \underset{\vec{a}}{\vec{x}} \quad = \quad \lambda \underset{a_0}{x_0} \, \lambda \underset{a_1}{x_1} \cdots \lambda \underset{a_n}{x_n} \tag{5.3}$$

Furthermore, for any type b, the type expression $\langle\!\langle \vec{a}, b \rangle\!\rangle$ represents the type of a (curried) function taking an arbitrary number of arguments, of arbitrary type, whose body is of type b.

$$\langle\!\langle \vec{a}, b \rangle\!\rangle \quad = \quad \langle\!\langle a_0, \langle\!\langle a_1, \langle\!\langle \cdots, \langle\!\langle a_n, b \rangle\!\rangle \cdots \rangle\!\rangle\rangle\!\rangle\rangle\!\rangle \tag{5.4}$$

Hence the Argument Raising and Lowering rules that Hendriks proposes are generic with respect to the number of arguments expected by the constituent α; the new denotation is expressed in terms of the old one, and the arguments that are not being raised (lowered) are merely accepted by the new denotation and passed in black-box fashion—without manipulation—into the old denotation. This trick is quite useful, and we will make use of it ourselves in the next chapter.

5.3 Meanings and derivations

Plural things must on *some* level be sets of things. In Schwarzschild [12] they are sets of entities; others add some group/sum machinery that makes them not the same as the sets we use to denote predicates. For this work, we will talk about them in the set terminology (and use {} notation to represent them), though there is very little that hinges specifically on this choice.

5.3.1 Nouns

Just as they always have, nouns denote sets of entities, of type et. For running examples, we will grant our universe two nouns, *kid* and *story*, each with three members Alex, Beth, and Chris, and Othello, Hamlet, and King Lear. Thus:

$$[\![\text{kid}]\!]_{et} \quad = \quad \{a, b, c\} \tag{5.5}$$

$$[\![\text{story}]\!]_{et} \quad = \quad \{o, h, k\} \tag{5.6}$$

In general, noun phrases are now of type et. (Generalised quantifiers will be handled below.) Assembling them from conjoined rigid designators (i.e. proper nouns) is straightforward set union. The determiner *the* selects the salient *subset* of an et noun, rather than the salient *member*. Bare plurals fall naturally out of the system—the native type of nouns is now et, so no modification is necessary.

5.3.2 Numbers

Numbers are of type ⟨*et*, *ett*⟩—they take nouns and return those members of the power set of their argument that have the appropriate cardinality.

$$\underset{etett}{[\![N]\!]} = \underset{et}{\overset{noun}{\lambda P}} \left[\underset{et}{\lambda \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge |\mathrm{X}| = [\![N^\star]\!]\right] \right] \tag{5.7}$$

This gives us the denotations

$$[\![\text{two stories}]\!]_{ett} = \{oh, ok, hk\} \tag{5.8}$$

$$[\![\text{three kids}]\!]_{ett} = \{abc\} \tag{5.9}$$

Note that there seem to be a few pseudo-numbers of this type: *several* and *many*, for instance. Crucially, though, *some* is *not* such a thing.

These sorts of numeric phrases are a little bit funny: since an NP is *et* and a GQ is *ettt* (see below), the *ett* numeric phrase seems like it doesn't fit anywhere. However, this form is needed at least in binominal *each* phrases, and possibly also in certain predicative situations (e.g. "Alex and Beth were two kids who. . . "). In cases where the numeric phrase is by itself going to be a subject or an object, we need to convert it into a generalised quantifier:

$$\underset{\langle ett,\, ettt\rangle}{\text{numlift}} = \underset{ett}{\overset{num\ phr}{\lambda \mathrm{P}}} \left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\underset{et}{\exists \mathrm{Y}} \left[\mathrm{P}(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right] \right] \right] \tag{5.10}$$

This is, perhaps unsurprisingly, very much like the denotation of the determiner *some* given below; it performs a similar sort of task.

Also worth noting is the fact that the denotation we give for numbers requires that the cardinality of a set be *equal* to a certain number. What, then, is the difference between ⟦two⟧ and ⟦exactly two⟧? The former merely asserts that something is true of two things. It may be true of more (and often is), assuming the context allows that. The latter, however, asserts that something is true of two things *and not of more than two.* As such, it seems that some sort of compound assertion is being made; see Section 7.1.3 for further discussion.

While it is possible for number phrases to appear by themselves as noun phrases, it seems clear that there must be some unary rule in the grammar to permit it—nearly every determiner can combine with a number phrase to make a full noun phrase.

The two kids slept. (5.11)

No two kids slept. (5.12)

Some two kids slept. (5.13)

are all valid, along with others besides. In addition, we have the evidence that *only* number phrases can appear as the Dist phrase in a binominal *each* construction. Hence, we feel that setting aside a different category for them is justified (and this in turn licenses having a separate type).

A slight wrench in the works is the fact that

*No several kids slept. (5.14)

*Some several kids slept. (5.15)

are bad. Nevertheless, this may arise from some pragmatic concerns involving the inexactness of the cardinality of *several*; after all, "the several" *is* valid, at least in certain contexts.

5.3.3 Verbs

If we change the meaning of noun phrases to be sets of entities, we will obviously also need to modify the denotation of verbs and verb phrases to suit.

Intransitive predicates

In a simpler system with no plurals, we would represent intransitive verbs as being predicates of type *et*. A first stab at introducing plurals might have us retaining these as the "base meanings" of verbs, with some transformation we could apply to derive the plural form. This works adequately for predicates that are fully distributive: if "Alex slept" and "Beth slept" are both true, then "Alex and Beth slept" is also true. The entailment runs in the reverse direction as well. However, with non-distributive predicates, this equivalence does not hold: "Alex lifted a piano" and "Beth lifted a piano" do entail "Alex and Beth lifted a piano", but the reverse entailment does not hold. "Alex met" doesn't even make sense, while "Alex and Beth met" is perfectly well-formed and meaningful.

Yoad Winter [14] introduces a mechanism to deal with this problem: he assumes that some verbs (like *sleep*) are of type *et*, while others (like *meet*) are of type *ett*, and then gives ways of shifting and lifting types to work out in the various cases.

We would argue that this is not the right way to go about the problem. Rather, *all* verbs should be of the "set predicate" type—*ett*. Then the fact that some such verbs are distributive becomes an analytical property of the meaning of those verbs, rather than anything that particularly plays out in the syntax or type system. We will use the term "fully distributive" to refer to a predicate P for which the following holds true:

$$\forall \mathrm{X}_{et}\,[\mathrm{P}(\mathrm{X}) \rightarrow \forall x_e \in \mathrm{X}\,[\mathrm{P}(x)]]$$

This is essentially the system of Scha [11], described above; the domain of *all* verb phrases include plural individuals. The distributivity of a predicate is a meaning postulate—living entirely in the semantics—rather than a syntactic or syntax-driven mechanism. As evidence for this (and a preview of things to come), we note that any reasonable denotation for *each* will need to control the distributivity of its Relation; any syntax-controlled distributivity mechanism would presumably not be available at that level of semantic processing.

Now that all (intransitive) verbs have effectively been made plural, how do we apply them to singular subjects? Two methods suggest themselves, but they are equivalent: either the (type *e*) subject needs to be raised into a singleton set (of type *et*), or the plural predicate (*ett*) needs to be lowered into a singular one. Winter prefers the latter (his *sg* operator), but we will assume the former, as it simplifies the framework: verbs have only one type, and noun phrases have only one type.

Transitive predicates

In the same way we raised *et* predicates to *ett* predicates, we will raise the *eet* transitive predicates to *etett* predicates. At first glance, this causes a difficulty: how do we decide on the *etett* denotation of the so-called "distributive" predicates? Well, we don't. While we might still think of a simple *eet* version of (some) transitive verbs, if the *etett* version is taken to be the base meaning, then the relationship between the two can be defined on a per-verb basis. In other words, we have moved this work out of the compositional semantics and into the lexical semantics.

Number agreement

We have been somewhat cavalier about dismissing the singular denotations as derived forms. This is not exactly true; there may be some meaning difference between, for example, *kid* and *kids*; but crucially, we believe that they are of the same semantic type. As evidence, we can look at languages that have dual or paucal number in addition to singular and plural; unlike singular noun phrases (which could *a priori* be either individuals of type *e* or singleton sets of type *et*), NPs of dual number must (on the type level) have the same sort of representation as those of plural number. Nevertheless, in these languages the distinction between dual and plural is maintained, so the human brain is clearly capable of and at least moderately predisposed towards making a morphological distinction based solely on the cardinality of a set.

For further evidence of a single type for singular and plural, consider numbers. Phrases involving the number *one* are syntactically distributed like phrases involving the number *two* or any other number; in particular, the object of a binominal *each* can be singular or plural *as long as it involves a number*. If singular and plural were fundamentally different, it seems that any semantics that accounts for this fact would have to be odd at best—*one* and *two* would have completely different types and meanings.

We will further speculate on what the semantic differences might be in Section 5.3.5, after we have introduced quantification.

5.3.4 Quantification

The foregoing sections lay out a type system that works just fine for simple subjects like "Alex and Beth" or "the kids", but how does this interact with quantifier phrases? If the morphological decision is made on the cardinality of a set, how is the decision made if there is no set to judge, as with "every kid" or "all kids"? How is the verb agreement determined in "no kid sleeps" as versus "no kids sleep"?

Basic ideas

Traditionally, quantifiers operated on nouns (of type *et*) and verbs (also of type *et*); now that verbs have been raised to *ett*, the types and meanings of quantifiers will change slightly, losing some of the symmetry they previously enjoyed:

$$[\![\text{some}]\!]_{\langle et, \langle ett, t\rangle\rangle} = \overset{noun}{\underset{et}{\lambda P}} \left[\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}} \left[\underset{et}{\exists \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge \mathrm{Q}(\mathrm{X})\right]\right]\right] \tag{5.16}$$

$$[\![\text{no}]\!]_{\langle et, \langle ett, t\rangle\rangle} = \overset{noun}{\underset{et}{\lambda P}} \left[\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}} \left[\neg\underset{et}{\exists \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge \mathrm{Q}(\mathrm{X})\right]\right]\right] \tag{5.17}$$

This yields the following reasonable denotations. We provide more complete derivations for these definitions in Appendix B.

$$[\![\text{some stories}]\!]_{ettt} = \overset{vb}{\underset{ett}{\lambda \mathrm{Q}}} \left[\exists \mathrm{X}_{et} \left[\mathrm{X} \subseteq \{o, h, k\} \wedge \mathrm{Q}(\mathrm{X})\right]\right] \tag{5.18}$$

$$[\![\text{no kids}]\!]_{ettt} = \overset{vb}{\underset{ett}{\lambda \mathrm{Q}}} \left[\neg\exists \mathrm{X}_{et} \left[\mathrm{X} \subseteq \{a, b, c\} \wedge \mathrm{Q}(\mathrm{X})\right]\right] \tag{5.19}$$

$$[\![\text{no kids sleep}]\!]_{t} = \neg\exists \mathrm{X}_{et} \left[\mathrm{X} \subseteq \{a, b, c\} \wedge [\![\text{sleep}]\!](\mathrm{X})\right] \tag{5.20}$$

When a generalised quantifier appears somewhere in the sentence other than subject position, the constituent it is argument to will be expecting an NP, of type *et*, so we must lift it to accept a GQ, of type *ettt*. Here we make use of the Argument Raising rule proposed by Herman Hendriks [3], discussed above in Section 5.2.4. Conveniently, his rule is expressed so generally that it applies without modification in our type system. However, we are not currently making use of any intensional definitions, and in this work it will be only used to convert *et* arguments to *ettt*, so we give a slightly less general version of his rule here:

$$\text{arglift} = \underset{\langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle}{\lambda \mathscr{R}} \left[\underset{\vec{b}}{\lambda \vec{B}} \underset{ettt}{\lambda \mathscr{P}} \underset{\vec{a}}{\lambda \vec{A}} \left[\mathscr{P} \left(\underset{et}{\lambda \mathrm{Y}} \left[\mathscr{R}(\vec{B})(\mathrm{Y})(\vec{A}) \right] \right) \right] \right] \tag{5.21}$$

As with the original rule, it can be applied to any denotation expecting zero or more arguments, then one of type *et*, then zero or more further arguments, finally returning a proposition of type *t*; the noted *et* argument is then the one that is lifted to expect a GQ. In the usual case of a transitive verb lifting its object argument, this further reduces to

$$\text{objlift} = \overset{verb}{\underset{\langle et, \langle et, t\rangle\rangle}{\lambda \mathscr{R}}} \left[\overset{obj}{\underset{ettt}{\lambda \mathscr{P}}} \left[\overset{sbj}{\underset{et}{\lambda \mathrm{X}}} \left[\mathscr{P} \left(\overset{obj}{\underset{et}{\lambda \mathrm{Y}}} \left[\mathscr{R}(\mathrm{Y})(\mathrm{X})\right] \right) \right] \right] \right] \tag{5.22}$$

This enables us to write things like "Chris read some stories":

$$[\![\text{read some stories}]\!]_{ett} = \overset{sbj}{\underset{et}{\lambda \mathrm{X}}} \left[\exists \mathrm{Z} \left[\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(X)\right]\right] \tag{5.23}$$

$$[\![\text{Chris read some stories}]\!]_{t} = \exists \mathrm{Z} \left[\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(\{c\})\right] \tag{5.24}$$

or "Bill read no stories":

$$[\![\text{read no stories}]\!]_{ett} = \overset{sbj}{\underset{et}{\lambda \mathrm{X}}} \left[\neg\exists \mathrm{Z} \left[\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(X)\right]\right] \tag{5.25}$$

$$[\![\text{Bill read no stories}]\!]_{t} = \neg\exists \mathrm{Z} \left[\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(\{b\})\right] \tag{5.26}$$

The conservativity problem

Generalised quantifiers have been shown to have a number of properties with regards to their lexical meaning; the most well known is the conservativity property. In Appendix A we address the question of how to modify these rules to fit our new type system.

5.3.5 Speculation on morphological number and quantifiers

We alluded earlier to a belief that—although carrying the same type—singulars and plurals may have slightly different lexical denotations. An excellent example of the problem can be seen if you consider that "some kid sleeps" and "some kids sleep" are both valid, *and mean different things*. How do we reconcile that?

Consider again the simple case, "Beth sleep(s)." The subject we represent as $\{b\}$, the predicate as $[\![\text{sleep}]\!]$. We need the verb to appear in the singular because its subject is a singleton set.

A slightly more complex case: "The kid(s) sleep(s)." In the case that there is a single contextually salient kid, "the kid(s)" marks a singleton set, thus its head (*kid*) and the verb it serves as argument to (*sleep*) will need to both be in the singular: "The kid sleeps." On the other hand, if there are two or more salient kids, *the* will pick out that larger set, causing the other two words to be plural: "The kids sleep." In either case, the form that is correct is dictated by the context, and for any given context only one will work.

Now we return to the GQ case: "Some kid(s) sleep(s)." While the denotation of the verb remains the same ($[\![\text{sleep}]\!]$), the denotation of the subject is more complex now ($\lambda \underset{ett}{\text{Q}} [\exists \text{X} [\text{X} \subseteq \{a, b, c\} \wedge \text{Q}(\text{X})]]$), and in particular, it is no longer simply a set we can judge to be singleton or not.

Lurking in the denotation, however, there *is* a place where sets can be analysed for cardinality: when X is consumed by the verb denotation Q. As before, if the argument of the verb is a singleton, the verb and the head of the subject are marked as singular; otherwise they are marked as plural. Now that either is contextually possible, the morphology is not just a redundant marking; the morphological choice made by the utterer serves as an additional assertion about the meaning of the proposition.

Formally, we can say that the morphology adds a term to the verb as follows:

$$\text{Q}_{\text{sg}} = \lambda \overset{sbj}{\underset{et}{\text{X}}} \left[\text{Q}_{\text{bare}}(\text{X}) \wedge |\text{X}| = 1\right] \quad (5.27)$$

For the plural, we will stipulate for now that it is underlyingly *unmarked* for number, and that the $|\text{X}| > 1$ is implied through Gricean principles. Our reason for this is discussed below.

$$\text{Q}_{\text{pl}} = \lambda \overset{sbj}{\underset{et}{\text{X}}} \left[\text{Q}_{\text{bare}}(\text{X})\right] \quad (5.28)$$

This allows us to derive the denotations we'd want:

$$[\![\text{Some boy sleeps}]\!] = \exists \underset{et}{\text{X}} [\text{X} \subseteq \{a, b, c\} \wedge [\![\text{sleep}]\!](\text{X}) \wedge |\text{X}| = 1] \quad (5.29)$$

$$[\![\text{Some boys sleep}]\!] = \exists \underset{et}{\text{X}} [\text{X} \subseteq \{a, b, c\} \wedge [\![\text{sleep}]\!](\text{X})] \quad (5.30)$$

In the case of the verb *sleep*, the plural case entails the singular, but this is due to the fully distributive nature of *sleep*. With a verb such as *meet*, the entailment does not hold.

The reason we wished above to make the plural unmarked for cardinality is to make the 'no' case work. In the singular, it works fine:

$$[\![\text{No boy sleeps.}]\!] \quad = \quad \neg\exists \underset{et}{\mathrm{X}} \,[\mathrm{X} \subseteq \{a, b, c\} \,\wedge\, [\![\text{sleep}]\!](\mathrm{X}) \,\wedge\, |\mathrm{X}| = 1] \tag{5.31}$$

The plural case only works properly if the denotation is unmarked:

$$[\![\text{No boys sleep.}]\!] \quad = \quad \neg\exists \underset{et}{\mathrm{X}} \,[\mathrm{X} \subseteq \{a, b, c\} \,\wedge\, [\![\text{sleep}]\!](\mathrm{X})] \tag{5.32}$$

Had the $|\mathrm{X}| > 1$ been part of the denotation, then the "no boys sleep" case would have been true even in the case where exactly one boy slept, which seems incorrect.

Chapter 6

An analysis of binominal *each*

Having established a working and consistent type system, we are now ready to discuss the lexical meaning of *each*, its lexical distribution patterns, and how it composes semantically with the rest of the sentence.

6.1 Lexical meaning and the basic case

Where most verbs can only assert things collectively of the subject,[1] propositions involving *each* must be true individually of each member of the subject set. This suggests the draft denotation shown in (6.1).

$$[\![\text{each}]\!] \quad = \quad \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \underset{et}{\mathrm{X}}\,[\mathrm{arglift}(\mathscr{R})(\mathrm{numlift}(\mathrm{P}))(\{x\})]\right]\right]\right] \tag{6.1}$$

We can, of course, substitute the definitions for the arglift and numlift operators, yielding the lexical denotation for *each* in (6.2).

$$[\![\text{each}]\!] \quad = \quad \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right]\right] \tag{6.2}$$

This can be compared to Zimmerman's definition, given earlier as (4.1) and repeated here with cosmetic modification as (6.3).

$$[\![\text{each}]\!] \quad = \quad \overset{obj}{\underset{Et}{\lambda \mathrm{P}}}\left[\forall \underset{e}{x} \in \overset{sbj}{\underset{E}{\mathrm{X}}}\left[\exists \underset{E}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \overset{vb}{\underset{Eet}{\mathscr{R}}}(x, \mathrm{Y})\right]\right]\right] \tag{6.3}$$

After the notational differences are removed, it should be clear that the definitions only significantly differ in their types—it actually isn't entirely clear what the types in Zimmerman's system are meant to be, but this is our best guess. Certainly the main thrust of the definition is the same.

[1] There may be further entailments—i.e. an assertion made of subsets or members of the subject set—but these entailments are properties of specific verbs, and not of the semantic system.

Indeed, at least in the basic case, the definition in (6.2) does exactly what we want. The denotation of the VP of the canonical (Full derivations, as before, are given in Appendix B.)

$$[\![\text{read two } \underset{ett}{\text{books}} \text{ each}]\!] \quad = \quad \lambda \overset{rng}{\underset{et}{\mathrm{X}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} \left[\mathrm{Y} \subseteq [\![\text{books}]\!] \wedge |\mathrm{Y}| = 2 \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\}) \right] \right] \right] \tag{6.4}$$

We could paraphrase the body as "for every member x of X, there is a batch of books, of size 2, that x has read." The upward entailment (that x could have read more than two books) is a property of the verb—if one person had in fact read three books, then the two-book subsets would also be in the set of things that person had read.

An important feature of this denotation for *each* is that it accounts for the cardinal restriction: the Dist phrase must be a NumP; anything else will not only cause a category mismatch, but a type mismatch as well.

The plurality constraint on the Range phrase is not handled by the given denotation. It could be asserted as part of the denotation—by inserting a term $|\mathrm{X}| > 1$ prior to the universal quantification—but this is purely stipulative, and we regard it as inelegant. It is, however, not inconsistent with any of the proposals in this chapter, so it may indeed be exactly what is needed. Nevertheless, we will omit this term from our derivations.

6.1.1 A CG framework

We have chosen to use a Categorial Grammar as our syntactic framework for analysis of the binominal *each* construction. Categorial Grammars of various denominations have a long history dating back to the first half of the 20th century, but it was Montague [8] who first integrated the CG syntactic frameworks with the older typed lambda calculus in the way that is now familiar. Its chief advantage contributing to its use in this work is this tight integration between the syntactic and semantic system. The category maps directly to the type, in a many-to-one mapping, and the syntactic structure of the sentence dictates the semantic composition. Syntactically, two sisters in the structure will be 'functor' and argument—the denotation of the functor will be a function that takes the denotation of the argument and returns a value. This returned value then becomes the denotation of the mother node. Occasionally, unary rules apply, accepting a constituent of a certain type and returning another. Crucially, though, we need not map the structure to some underlying form before trying to interpret the sentence; it is the surface structure that dictates the semantic interpretation.

As discussed in Section 6.2.1, we will be including wrap in our framework; category slashes that call for a wrapped constituent will be indicated with a subscript W ($/\!\!/_W$). For added clarity, we will also annotate left and right slashes in this fashion ($/_L$, $/_R$)—following the usage in [4] and elsewhere—rather than using the left-leaning and right-leaning slashes (\, /) common in the CG literature. In all cases, the category label for the functor will be the category of the mother, then a slash (usually annotated for direction), then the category of the argument.

To diagram our analyses, we will use tree and tree-like diagrams, labelled with category labels. Figures 6.1 and 6.2 on the facing page show two typical CG derivations, the first of a vanilla subject-verb-object sentence and the latter of a sentence with a generalised quantifier in object position.

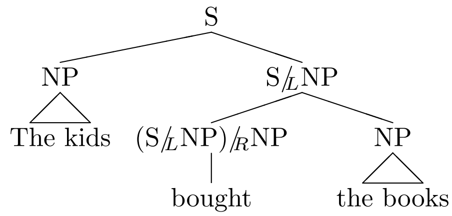

Figure 6.1: A CG-style derivation of a simple SVO sentence

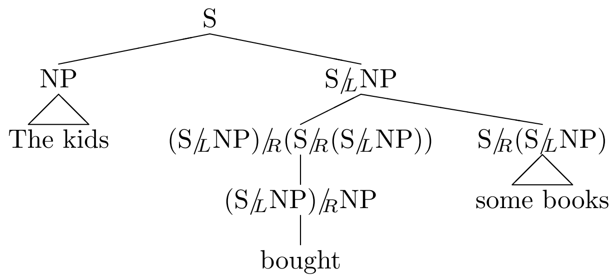

Figure 6.2: A CG-style derivation with a GQ as direct object

Now, let us consider what the syntax needs to look like for the canonical case given in (2.1) and repeated here as (6.5):

The kids bought three books each. (6.5)

The word *each* needs to take three arguments—the Range, the Relation, and the Dist. These are, respectively, a noun phrase, a transitive verb, and a numeric phrase; and let us assume that these are the most basic types available for their respective roles in the binominal *each* construction. Then the type for *each* can be given as $((S/_L NP)/_L((S/_L NP)/_R NP))/_L \mathrm{NumP}$, and the straightforward derivation will look something like that shown in Figure 6.3.

6.2 Ditransitives

We have shown how the binominal *each* construction works on normal transitive verbs. What, then, of ditransitives? Consider again the case where an indirect object is present, plural, and acting as

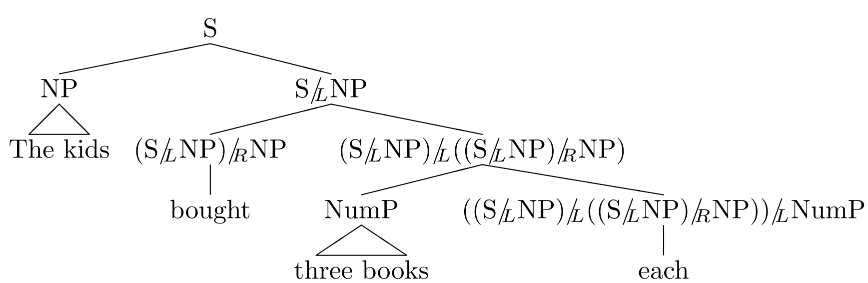

Figure 6.3: A CG-style derivation for (2.1)

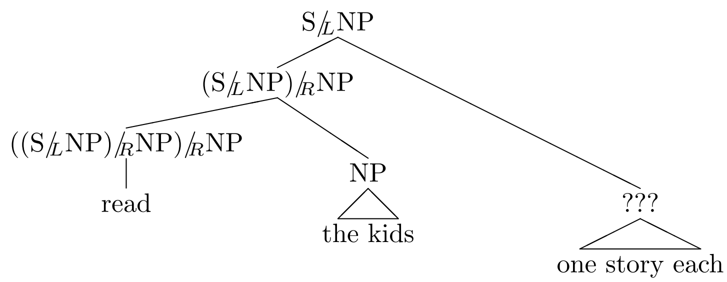

Figure 6.4: A naïve analysis of a ditransitive construction

the Range phrase of the *each* construction:

Alex read the kids one story each. (6.6)

Here the verb is of the wrong type to be taken as an argument of *each*, at least by the definition we gave earlier. One possible tactic would be to make use of the structural completeness property of (some) Categorial Grammar models: rebracket the sentence so that "Alex read" itself becomes a constituent, a two place relation awaiting a direct object and an indirect object, which we can use as the Relation argument to the *each* construction.

However, if we hinge our analysis on the ability to make a single constituent of "Alex read", we will experience difficulties when presented with sentences such as

Alex read the kids one story each and smiled. (6.7)

Here we would expect "read. . . each" to be a constituent verb phrase, to conjoin with the intransitive verb phrase "smiled". Hence we would like to find an analysis where *each* can take ditransitive verbs directly.

6.2.1 A short digression on the question of wrap

A number of different theories exist as to how to parse ditransitive verbs. One is that the verb first combines with the innermost object (here the indirect object, "the kids")—this preserves a nice tree structure, and might be represented as in Figure 6.4.

This is convenient, but probably not accurate. Other arguments have been given against this analysis [4], but in the current case, we note that a compositional analysis that accounts for the meaning of *each* will need to have the Relation and Range separately combining with constituents containing the *each*, but in Figure 6.4, the Range and Relation combine first with each other. We suppose that it might be possible to make a compositional analysis accounting for this fact, but it would require a great deal of additional machinery.[2]

[2]For those who remain unconvinced of the necessity of wrap as such, they should find that the same methods used to simulate wrap in other circumstances should work here. While we speak of the *each*-phrase as being the functor and the verb the argument, the verb could be lifted over the *each*-phrase, yielding the same syntactic structure as a basic ditransitive; the wrap simulation can then apply in just the same fashion as it usually does.

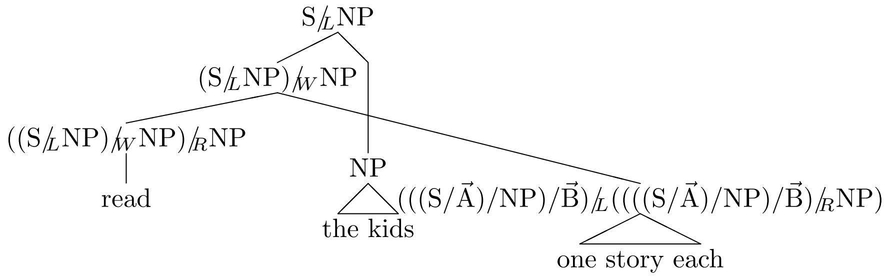

Figure 6.5: Preferred analysis of ditransitive with indirect object as Range

Rather, we need a theory that permits the verb to first combine with the rightmost argument—here the direct object—and only then combine with the inner argument, and finally the subject. There are two prevailing theories in this regard: wrap and movement. The wrap theory posits that the parse structure is not strictly a tree, and permits discontiguous constituents, being able to take the (discontiguous) constituent comprised of the verb and its first argument and 'wrap' it around the second argument, as shown in Figure 6.5. The movement theory instead suggests that the underlying form has the lower constituent (the direct object, "one story each") adjacent to the verb, but the surface form moves this to the end of the sentence, leaving a trace. We feel that the wrapped "trees" are somewhat more concise and easy to understand, so we will be using them henceforth, but the wrap-vs.-movement decision is largely orthogonal to the questions under consideration.

6.2.2 Binominal *each* with ditransitives

In Section 5.2.4, we discussed the genericity of Herman Hendriks' type shifting rules with respect to number of arguments and exact argument structure. Here we will use a similar trick (and the same notation) to generalise our *each* denotation for use with three-place verbs as well as two-place verbs.

The first argument to the verb corresponds to the Dist phrase; this argument position will remain et. (This is not the type of the Dist phrase, of course, but the *each*-phrase expects a relation that expects a noun phrase first.) Of the remaining arguments to the verb, either one can be the Range phrase. As such, we need to allow a flexible group of arguments to be taken either before or after the Range phrase; using the arrow notation for these gives us the following *each* denotation:

$$[\![\text{each}]\!] = \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}}\left[\overset{rng}{\lambda \underset{\vec{b}}{\vec{B}} \lambda \underset{et}{\mathrm{X}} \lambda \underset{\vec{a}}{\vec{A}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\right] \tag{6.8}$$

Note that this is virtually identical to the previous version in (6.2) on page 33, except for the addition of the dummy variables $\vec{A}$ and $\vec{B}$.

Unlike Hendriks, whose categories were invariant amidst the flexible shifting within the type system, our category must also vary in a similar fashion. We have thus extended the arrow notation to also work with categories: for any category X, the category $(\mathrm{X}/\vec{\mathrm{A}})$ represents a syntactic constituent that will combine with some arbitrary number of other constituents, of arbitrary categories,

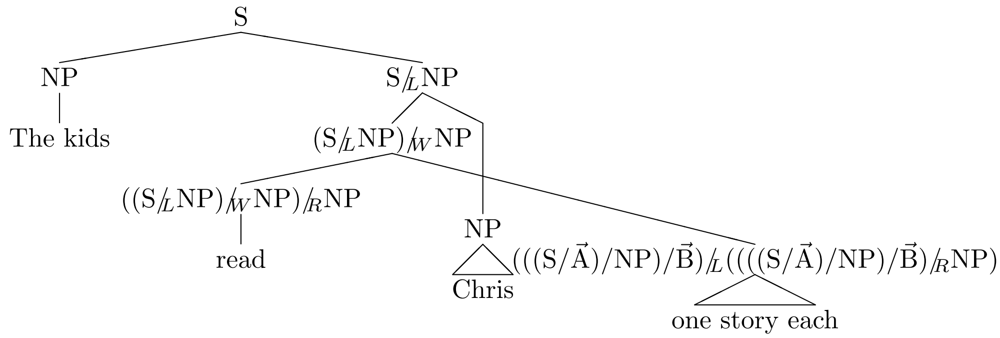

Figure 6.6: Analysis of ditransitive with subject as Range

eventually yielding a final outer constituent of category X.

$$(X/\vec{A}) \quad = \quad (((\cdots(X/A_n)/\cdots)/A_1)/A_0) \tag{6.9}$$

Thus the new category for *each* can be written $((((S/\vec{A})/NP)/\vec{B})/_L((((S/\vec{A})/NP)/\vec{B})/_R NP))/_L NumP$.

Thus armed, we can now return to our analysis of the ditransitive cases. For the sentence in (6.6), we now have an analysis (shown in Figure 6.5 on the previous page) that yields a constituent verb phrase whose meaning is as follows:

$$\lambda \underset{et}{A} \left[\forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{Y} \left[[\![\text{one story}]\!](Y) \wedge [\![\text{read}]\!](Y)(\{x\})(A)\right]\right]\right] \tag{6.10}$$

This combines with the subject ("Alex") to give the desired meaning for the sentence.

In a similar fashion we can address the case where the Range is the subject, as in the sentence

The kids read Chris one story each. (6.11)

The analysis is in Figure 6.6 and the resultant denotation is as follows:

$$\forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{Y} \left[[\![\text{one story}]\!](Y) \wedge [\![\text{read}]\!](Y)(\{c\})(\{x\})\right]\right] \tag{6.12}$$

Finally, ditransitives with multiple plural arguments give rise to an ambiguous reading of the *each* construction:

Alex and Loren read the kids one story each. (6.13)

Here, we could have the subject as Range, making two stories, one read by Alex and one by Loren; or we could have the indirect object as Range, making one story per kid, all of which were collectively read by Alex and Loren. Formally,

$$\forall \underset{e}{x} \in \{a, l\} \left[\exists \underset{et}{Y} \left[[\![\text{one story}]\!](Y) \wedge [\![\text{read}]\!](Y)([\![\text{the kids}]\!])(\{x\})\right]\right] \tag{6.14}$$

$$\forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{Y} \left[[\![\text{one story}]\!](Y) \wedge [\![\text{read}]\!](Y)(\{x\})(\{a, l\})\right]\right] \tag{6.15}$$

6.3 GQs as Range phrases

The examples we have given so far have used simple noun phrases as the Range phrase of the *each* construction. The semantics work out just as well when generalised quantifiers occupy those positions, provided that they are plural. The denotation for the near-canonical example in (2.10) is as follows:

$$\begin{aligned}&[\![\text{Some kids bought three books each.}]\!]\\ =\ &\underset{et}{\exists \mathrm{Z}}\left[\mathrm{Z} \subseteq [\![\text{kids}]\!] \wedge \underset{e}{\forall x} \in \mathrm{Z}\left[\underset{et}{\exists \mathrm{Y}}\left[[\![\text{three books}]\!](\mathrm{Y}) \wedge [\![\text{bought}]\!](\mathrm{Y})(\{x\})\right]\right]\right]\end{aligned} \tag{6.16}$$

A NumP in subject position will be converted into a GQ, yielding a similar analysis for (2.11):

$$\begin{aligned}&[\![\text{Two kids bought three books each.}]\!]\\ =\ &\underset{et}{\exists \mathrm{Z}}\left[[\![\text{two kids}]\!](\mathrm{Z}) \wedge \underset{e}{\forall x} \in \mathrm{Z}\left[\underset{et}{\exists \mathrm{Y}}\left[[\![\text{three books}]\!](\mathrm{Y}) \wedge [\![\text{bought}]\!](\mathrm{Y})(\{x\})\right]\right]\right]\end{aligned} \tag{6.17}$$

Denotations for the ditransitive case require no additional mechanisms, simply applying the usual argument lift rule on the verb:

$$\begin{aligned}&[\![\text{Alex gave some girls four presents each.}]\!]\\ =\ &\underset{et}{\exists \mathrm{X}}\left[\mathrm{X} \subseteq [\![\text{girls}]\!] \wedge \underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[[\![\text{four presents}]\!](\mathrm{Y}) \wedge [\![\text{gave}]\!](\mathrm{Y})(\{x\})(\{a\})\right]\right]\right]\end{aligned} \tag{6.18}$$

6.4 Prepositional Relations

We have seen that our denotation for the word *each* works in the cases where the Relation is a verb, either transitive or ditransitive. We will now address the case where it is a preposition.

Consider the sentence given in example (2.57), repeated here as (6.19):

Mickey wears gloves with four fingers each. (6.19)

The meaning of this should be clear, and it should likewise be clear that in this *each* construction, "gloves" is the Range and "four fingers" is the Dist; and thus that the Relation is simply that expressed by the preposition *with*, which we might call a genitive relationship.

6.4.1 On prepositions

The standard CG analysis of prepositional constructions is fairly straightforward, but we will review it briefly here. A preposition is a word that takes an object and produces a phrase that acts as a modifier. The object is generally a noun phrase, and this is the only case relevant here. The prepositional phrase can modify a number of different things, but again, the only currently relevant variety is the kind that modifies nouns. Since the preposition accepts a noun phrase object (to its right), and produces a noun modifier (that modifies its left sister), we say that its category is $(\mathrm{N}/_{L}\mathrm{N})/_{R}\mathrm{NP}$. If the word *each* were not present in (6.19), we would diagram it as in Figure 6.7 on the next page.

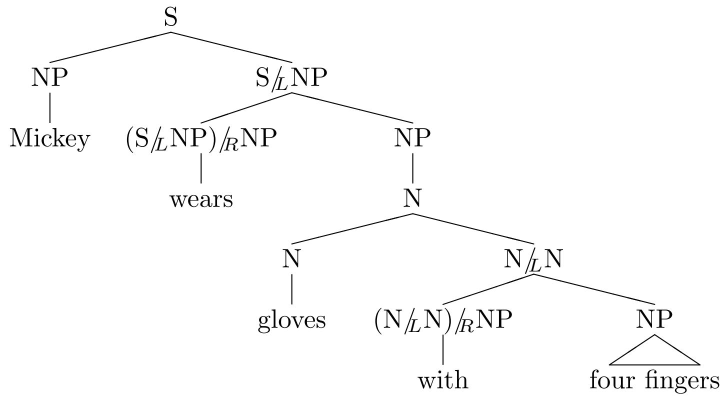

Figure 6.7: Traditional analysis of sentence with basic noun-modifying PP

The denotation of the PP in a traditional framework might look something like this:

$$[\![\text{with four fingers}]\!] \quad = \quad \lambda \underset{et}{\overset{noun}{\mathrm{X}}} \left[\lambda \underset{e}{x} \left[x \in \mathrm{X} \wedge [\![\text{`has' four fingers}]\!](x) \right] \right] \tag{6.20}$$

That's all fine and good, but it fails to account for pluralities in any meaningful way. Specifically, the "has four fingers" predicate is only applied to individual entities x, not to a whole group. A phrase like "jackets with buttons" clearly is inclusive of jackets with one button each; if we allow that morphology is more than simply cosmetic (see Section 5.3.5), then this provides evidence that the phrase "with buttons" is somehow applying to the, or a, group of jackets, rather than individual jackets one at a time.

In order to handle this idea without further complicating the type system, I introduce a new binding construct 'ℓ' to our formalism. The exact definition and ramifications of this construct are not the focus of this paper, so I will give a more informal sketch: ℓ constructs the largest and/or most relevant set that meets a certain set of conditions.[3] Using this notation, we will write our desired denotation for "with four fingers" as follows:

$$[\![\text{with four fingers}]\!] \quad = \quad \lambda \underset{et}{\overset{noun}{\mathrm{X}}} \left[\ell \underset{et}{\mathrm{Z}} \left[\mathrm{Z} \subseteq \mathrm{X} \wedge [\![\text{`have'}]\!]([\![\text{four fingers}]\!])(\mathrm{Z}) \right] \right] \tag{6.21}$$

In prose, we might write this as "from some set X (e.g. $[\![\text{gloves}]\!]$), extract the largest subset Z such that Z collectively have four fingers". We thus account for the notion that the prepositional phrase modifies the noun—restricting it to a constructed subset of its original denotation—via a predicate over the entire constructed subset.

This may at first seem like a great deal of work for little gain; why not just make such prepositional modifiers always distributive? This is a natural first reaction, since the default reading in most such cases is indeed distributive. However, we should recall that there are other binominal modifiers; in particular, consider the following example using *altogether*:

The kids with one blanket each were warmer than the kids with two blankets altogether. (6.22)

[3]Note the consequences for the expression type: unlike λ, where a variable of type *et* will yield a λ-expression of type *ett*, with ℓ a variable of type *et* will yield a resulting ℓ-expression of type *et*.

This might come in the context of a camping trip, where one batch of kids were well-prepared and another batch wasn't, having to share two blankets among (say) five kids. Crucially, it needs to be possible for the prepositional phrase to apply to the entire constructed subset, not just its individual members.

Unfortunately, our new denotation for prepositional phrases will not work directly with *each*. What we would *like* as a denotation for "with four fingers each" is something like

$$[\![\text{with four fingers each}]\!] \quad = \quad \overset{noun}{\underset{et}{\lambda \mathrm{X}}} \left[\underset{et}{\ell \mathrm{Z}} \left[\mathrm{Z} \subseteq \mathrm{X} \wedge \forall \underset{e}{z} \in \mathrm{Z} \, [[\![\text{have}]\!]([\![\text{four fingers}]\!])(\{z\})] \right] \right] \tag{6.23}$$

Here we have "for some set, extract the largest subset Z such that *each member* of Z individually has four fingers." However, if this whole subset-constructing structure is part of the lexical meaning of *with*, there is no way for the *each* denotation to insert the universal quantifier in the middle. To remedy the situation, we need to adjust the mechanics of prepositional phrases somewhat.

6.4.2 An intermediate PP denotation

We suggest that the type of a preposition, such as *with*, is not $(\mathrm{N}/_L\mathrm{N})/_R\mathrm{NP}$, but rather simply $\mathrm{PP}/_R\mathrm{NP}$. There is then a unary rule in the grammar $\mathrm{PP}\rightarrow(\mathrm{N}/_L\mathrm{N})$. Furthermore, while the type of an $\mathrm{N}/_L\mathrm{N}$ is clearly $\langle et, et\rangle$, we propose that the type of the new PP category is simply $\langle et, t\rangle$. The mechanism of the unary rule will be similar to what might be used for (intersective) adjectives, essentially encapsulating a sort of predicate modification rule.

Our motivation for this is, of course, that it will help us to explain the *each* construction. However, there is a further justification: the entire mechanism of constructing a largest subset that meets a certain condition is one that will need to be present in every modifier; but the basic meaning of the prepositional phrase is fundamentally predicative. Hence an identical mechanism would need to be present in the lexical denotation of every preposition, to convert from predicate to modifier; and as every good computer scientist knows, it is always a good idea to factor out common code.

There is possibly yet another justification: prepositional phrases can in general modify either nouns or verb phrases, which are not only different categories but also different types. It is just possible that a single PP:*ett* category can be converted into *either* an N:*et* modifier *or* a $\mathrm{S}/_L\mathrm{NP}$:*ett* modifier, via two different unary rules. We will discuss this idea further in Section 7.1.1.

Under this new schema, the PP "with four fingers" is a predicate, true of any group of entities that has four fingers (either collectively, or individually). The PP "with four fingers each" is likewise a predicate, true of any group whose members each have four fingers:

$$[\![\text{with four fingers}]\!]_{\mathrm{PP}} \quad = \quad \underset{et}{\lambda \mathrm{X}} \, [[\![\text{with}]\!]([\![\text{four fingers}]\!])(\mathrm{X})] \tag{6.24}$$

$$[\![\text{with four fingers each}]\!]_{\mathrm{PP}} \quad = \quad \underset{et}{\lambda \mathrm{X}} \, [\forall x \in \mathrm{X} \, [[\![\text{with}]\!]([\![\text{four fingers}]\!])(\{x\})]] \tag{6.25}$$

Now we need to write the denotation for our unary rule:

$$\mathrm{PP} \rightarrow \mathrm{N}/_L\mathrm{N} \quad = \quad \overset{PP}{\underset{ett}{\lambda \mathrm{P}}} \left[\overset{noun}{\underset{et}{\lambda \mathrm{X}}} \left[\underset{et}{\ell \mathrm{Z}} \, [\mathrm{Z} \subseteq \mathrm{X} \wedge \mathrm{P}(\mathrm{Z})] \right] \right] \tag{6.26}$$

Unfortunately, we're not done yet. While we have justified the need for PP modifiers to sometimes be collective, the system we have set up now *forces* a collective interpretation unless specifically overridden by the presence of *each*. The final $\mathrm{N}\not{\!L}\mathrm{N}$ denotation of "with four fingers" would be

$$\lambda \underset{et}{\overset{noun}{X}} \left[\ell \underset{et}{Z} \left[Z \subseteq X \wedge \exists \underset{et}{Y} \left[[\![\text{four fingers}]\!](Y) \wedge [\![\text{with}]\!](Y)(Z)\right]\right]\right] \tag{6.27}$$

—note that the existential quantification is *outside* the *with* predicate, which requires a collective reading. Since the distributive reading seems to be the default unless overridden, we would seem to have a problem on our hands. To solve it, we will need to change the type of prepositional phrases yet again.

6.4.3 Another type for PPs

In order to generate both a collective and a distributive meaning, the *with* predicate needs to have direct access to a generalised quantifier object—relying on a lift operator will always cause the quantification to occur outside of the application of the predicate. Thus the native type of *with* becomes $\langle ettt, \langle et, t\rangle\rangle$. This idea of letting a transitive predicate natively accept GQ objects is of course not new, dating back at least as far back as 1973, when Montague [8] made *all* transitive verbs take a 'term', or GQ, as object.[4] It is not unreasonable to extend this to prepositions, and it seems to be called for here.

Adopting this new type for *with*, then, yields the following meaning for the phrase:

$$\begin{aligned}
&[\![\text{with four fingers}]\!]_{\mathrm{N}\not{L}\mathrm{N}} \\
&\quad = \mathrm{PP}{\to}\mathrm{N}\not{\!L}\mathrm{N}\left([\![\text{with}]\!](\text{numlift}([\![\text{four fingers}]\!])) \right) \\
&\quad = \lambda \underset{et}{\overset{noun}{X}} \left[\ell \underset{et}{Z} \left[Z \subseteq X \wedge [\![\text{with}]\!] \left(\lambda \underset{ettt}{\overset{vb}{Q}} \left[\exists \underset{et}{Y} \left[[\![\text{four fingers}]\!](Y) \wedge Q(Y)\right]\right]\right)(Z)\right]\right]
\end{aligned} \tag{6.28}$$

This is not substantially different from our desired definition given above in (6.21). It does place the burden of ambiguity—between collectivity and distributivity—into the lexical denotation for *with*, which we will not try to give here.

Returning to the *each* case, we see that we have created a type conflict! The *each*-phrase is expecting a Relation whose first argument is *et*; but now our Relation is expecting an argument of type *ettt*. We will thus make use of Hendriks' Argument Lowering rule on the preposition, returning it to the type *etett* for use in the *each* construction.

A version of the lowering rule, with extensional arguments and made specific to our type system:

$$\text{arglower} = \underset{\langle \vec{b}, \langle ettt, \langle \vec{a}, t\rangle\rangle\rangle}{\lambda \mathscr{R}} \left[\lambda \underset{\vec{b}}{\vec{B}} \lambda \underset{et}{X} \lambda \underset{\vec{a}}{\vec{A}} \left[\mathscr{R}(\vec{B}) \left(\lambda \underset{ett}{P} [P(X)]\right)(\vec{A})\right]\right] \tag{6.29}$$

[4]He indicated earlier, in [7], that at least some verbs (e.g. *seek*) were not fully extensional and needed to take an intensional object. Hendriks [3] develops this and refers to certain verbs as 'intensional' even when working in an entirely extensional semantic framework—these verbs accept GQ objects even in their extensional form.

This lets us treat *with* exactly as we had been in the previous section; and lets *each* take it as its Relation argument. Indeed, the *each* case works out just as we had hoped:

$$
\begin{aligned}
&[\![\text{with four fingers each}]\!]_{\mathrm{N}/_{L}\mathrm{N}} \\
&\quad = \mathrm{PP}{\rightarrow}\mathrm{N}/_{L}\mathrm{N}\Big(\,[\![\text{each}]\!]([\![\text{four fingers}]\!])(\text{arglower}([\![\text{with}]\!]))\Big) \\
&\quad = \overset{noun}{\underset{et}{\lambda \mathrm{W}}}\Big[\underset{et}{\ell \mathrm{Z}}\Big[\mathrm{Z} \subseteq \mathrm{W} \wedge \forall \underset{e}{x} \in \mathrm{Z}\Big[\exists \underset{et}{\mathrm{Y}}\Big[[\![\text{four fingers}]\!](\mathrm{Y}) \wedge [\![\text{with}]\!]\Big(\underset{ett}{\lambda \mathrm{P}}[\mathrm{P}(\mathrm{Y})]\Big)(\{x\})\Big]\Big]\Big]\Big]
\end{aligned}
\tag{6.30}
$$

Full derivations are given in Appendix B. It is worth noting that the denotation for *each* is unchanged. Its category needs to change, though not significantly: the output type is no longer $\mathrm{S}/\vec{\mathrm{A}}$ (since a preposition's output type is PP, not S), but just $\vec{\mathrm{A}}$, yielding a final category for *each* of $(((\vec{\mathrm{A}}/\mathrm{NP})/\vec{\mathrm{B}})/_{L}((\vec{\mathrm{A}}/\mathrm{NP})/\vec{\mathrm{B}}))/_{L}\mathrm{NumP}$.

Could this all have worked without the extra PP mechanics? Quite probably, but it would not have been possible to have the prepositional phrase modifying an entire group. In order to discard the extras, one would need to find some other explanation for sentences like (6.22).

6.5 Restrictions

As noted in Sections 2.1.1 and 4.2.3, portions of the *each* construction have restrictions as to what sorts of values can fill them. Our system handles these restrictions to varying degrees.

6.5.1 Dist phrase cardinality

The cardinality of the Dist phrase is guaranteed through our creation of a new syntactic category NumP, and the fact that *each* requires its Dist argument to be of this category. Non-numeric Dist phrases (e.g. "several books", "many books") are explained by making words like *several* and *many* 'number-like' in the sense that they build NumPs in the same way that numbers do. Non-cardinal Dist phrases (e.g. "some book(s)", "the book(s)", "Othello") are correctly disallowed on the basis of not being convertible to NumPs.

6.5.2 Range phrase plurality

We mentioned briefly in Section 6.1 that a stipulative assertion could implement the plurality constraint by adding a term $|\mathrm{X}| > 1$ before the universal quantifier. In the case of "every kid" we then get a denotation something like

$$
\begin{aligned}
&[\![\text{Every kid bought three books each.}]\!] \\
&\quad \approx \forall \underset{et}{z} \in [\![\text{kid}]\!]\Big[\underline{|\{z\}| > 1} \wedge \forall \underset{e}{x} \in \{z\}\Big[\exists \underset{et}{\mathrm{Y}}[[\![\text{three books}]\!](\mathrm{Y}) \wedge [\![\text{bought}]\!](\mathrm{Y})(\{x\})]\Big]\Big]
\end{aligned}
\tag{6.31}
$$

However, this seems a bit strange: the underlined portion of the denotation merely makes the result *false*. To actually make it invalid, we would need to make this stipulation at a different level than the denotational semantics. The restriction may thus be fairly pragmatic. It almost certainly is not syntactic; morphological plurality is definitely insufficient, and in any case, there are sentences like (2.51), repeated below:

The kids' minds were occupied with needing to buy three books each. (6.32)

Passing a syntactic plurality restriction through this chain of verbal constructions would challenge even the most baroque of theories. The semantic or pragmatic restriction poses no such problem; our main argument against a semantic rule here is that it changes the role of the semantics from providing a meaning for valid sentences, to also helping determine the validity of those sentences. (This is not without precedent, of course; Zimmerman, for instance, uses the semantics in exactly this way.) Nevertheless, it seems clear that if one were to use the semantics to implement the plurality restriction, the same mechanism that blocks ordinary singular noun phrases from being the Range will also be able to block the bad GQs "every kid", "each kid", etc.; if the morphology works roughly as we speculate in Section 5.3.5, then "no kid" will also be blocked.

6.5.3 Clausemate

This restriction requires that the Range phrase be in the same clause as the Dist phrase (and Relation). Our system manages this in much the same way as other clausemate restrictions are implemented in CG: the default sentence construction will have the clause of category S and type t, and a subordinating conjunction expecting to take that category and type as argument; thus any syntactic mechanism will not be able to take the 'higher' portions of the sentence as argument. The problem with this is that CG's ability to freely rebracket the sentence causes this clausemate block to not work. However, we don't regard this as a problem in our system—since all CG frameworks have the same difficulty, we believe this is fully orthogonal to the *each* question.

Chapter 7

Conclusion

7.1 Unresolved issues

I have presented a system that handles the basic case and several of the more complex cases, but there remain a number of cases we have not explained, which will form the basis for future work in this area.

7.1.1 VP adjunct modifiers

It is in the case of adjuncts modifying the VP that we find a major inadequacy in the system. If we take a predicate that is not fully distributive, such as “lifted a piano”, we can certainly represent that meaning within our system. If Alex and Sasha (together) lift a piano, then we can give the denotation

$$[\![\text{lifted a piano}]\!] \quad = \quad \{\{a, s\}\} \tag{7.1}$$

This much is relatively straightforward. But when we attempt to modify the VP, a curious situation arises. Let’s say that Alex and Sasha weren’t able to lift the piano alone, but needed mechanical help, in the form of a few jacks they had lying around the garage:

Alex and Sasha lifted a piano with two jacks. (7.2)

The method for implementing the modification is not obvious. But maybe the word *with* can do the work for us, and it may seem we can thus wave it away as someone else’s problem (i.e. ‘not in the scope of this research’). However, the relevance comes when we consider the sentence

Alex and Sasha lifted a piano with two jacks each. (7.3)

$\vdash$ Alex and Sasha lifted a piano. (7.4)

$\nvdash$ Alex lifted a piano with two jacks. (7.5)

$\nvdash$ Alex and Sasha each lifted a piano with two jacks. (7.6)

This example demonstrates the extremely subtle modifications that need to be possible: the phrase "with two jacks each" needs to distribute itself in some fashion over each member of the subject, *without* making the verb phrase itself distribute!

It seems like handling this problem will require rather more in the semantic toolbox than we have discussed here. Even if we make use of the trick introduced in Section 6.4.2, of having an intermediate category PP—of whatever type—the problem is not immediately solved. I suspect that the key lies using this intermediate-category tactic in combination with something else, such as an event-based semantics, permitting the semantics to universally quantify the modifier to an event, then as part of the rule converting the PP into a verb phrase modifier, assign the event to the subject as a whole. In any case, though, a more complete explanation of prepositional modification will have to precede any further work in this direction.

7.1.2 Free noun phrases

Occasionally we can have free noun phrases in a sentence, acting as adjunct VP modifiers, as in

Alex and Sasha woke three times/three times each. (7.7)

These present difficulties any which way you look at them. First of all, assuming there is some unary rule converting a noun phrase (or rather, a GQ: "every time...") into a VP modifier, it can't apply before the *each* has taken the number phrase (or it will be of the wrong type), and it can't apply to the whole *each*-phrase, because *that* will be the wrong type. Essentially, there is no convenient Relation for *each* to take as its argument. If we posit a covert preposition preceding the free noun phrase, then we just reduce this to the problem of VP-modifying prepositional phrases discussed in the previous section.

7.1.3 Complex numbers

For dealing with numerical constructions, Yoad Winter [14] decided to assume that simple numbers " 'basically' denote predicates", while complex numbers are quantificational. It is certainly true that there seems to be more going on with numeric phrases like "at most three" and "between two and five". While we can come up with *ett* denotations for NumPs involving these phrases, the upwards entailment proceeding from both the verb and the numlift operator will make them not mean what was intended.

One solution would certainly be to just make them quantificational, as Winter (and others) do. However, that negates the value of having a separate NumP category—especially with regards to the *each* construction, which *can* take NumPs involving these complex numbers.

Perhaps an additional numlift operator would be in order. For all of the complex numbers, it is possible to specify the NumP with the same *ett* type, *if* the resulting GQ were more exclusive, having a component that read something like 'and not any sets of only [kid]s that aren't in this list'. Right now, the denotation of "three kids" is the set of all size-3 sets of kids; and the numlifted version is all those set predicates (i.e. verb phrases: *ett*) containing at least one member that is a size-3 set of kids. Nothing prevents those set predicates from also being true of other-size sets of kids.

On a phrase like "at most three kids", we can immediately come up with a plausible *ett* denotation: the set of all sets of kids, of size 3, 2, or 1 (or zero, but more on that in a minute). This denotation won't work with the existing numlift operator, but if we used a numlift operator that turned that into a quantifier of all those predicates containing at least one member of that set *and **no** kid-only members that **aren't** in that set*, then we'd have our complex numbers. This works equally well for "at least", "between X and Y", and—importantly—"exactly".

It's not exactly clear how best to implement this alternate numlift, however.

Somewhat independent of the complex number case is the notion of what to do about zero. It does act like a number in being able to participate in an *each*-phrase; but that's about it. It does not have the "at least" implication in its simple form; actually, it seems to be equivalent to *no*. As for a complex number like "between zero and four", it's difficult to say how to implement that, too: the natural first pass would be to make zero of something correspond to the empty set, but that isn't (and shouldn't be) part of the denotations of most verbs.

7.1.4 Intension and extension

The semantic framework developed in this work has been entirely extensional. This is more than adequate for the analyses we present, but fitting this into other work will require the ability to intensionalise denotations.

In particular, I suspect that the conversion of plural entities to an intensional form will be nontrivial. To start with, while the types for noun and noun phrase are the same in the extensional system (*et*), my first intuition is that they would be different intensionally: $\langle se, t\rangle$ for the nouns (as in the traditional **IL**), but $\langle s, et\rangle$ for the noun phrases—the *et* here actually represents a set of entities in a given time-and-world.

The ramifications of this distinction should prove interesting as they ripple through the system.

The type of a verb phrase would naturally be $\langle s, \langle\langle s, et\rangle, t\rangle\rangle$; then the type for the new PP category would be the same—a predicate over sets. But if the type for nouns is $\langle se, t\rangle$, then the type for N/$_L$N would have to be $\langle\langle se, t\rangle, \langle se, t\rangle\rangle$. How, then, to convert from PP to N/$_L$N? The conversion becomes nonobvious.

These difficulties are certainly not insurmountable, but they will require careful thought.

7.2 Contributions

7.2.1 Main contribution: Number Phrase

In this work, I developed a semantic framework that, among other things, included a separate category NumP for phrases involving a number or number-like word. This is justified by the fact that binominal *each* (as well as *apiece* and *altogether*) select specifically for such phrases. Further justification for a separate category can be found in the existence of phrases like “all three kids”, “the three kids”, “some three kids”, and so on. There is a slight catch in that “all several kids” and “some several kids” are bad (although “the several kids” is valid in certain contexts), but there definitely seems to be a separate syntactic distribution for these phrases.

7.2.2 Secondary contributions

I presented a semantic analysis of the binominal *each* construction that is *syntax-driven*. Previous work in this area has used analyses that are at least partially divorced from the surface structure of the sentence. My work is thus the first truly *compositional* analysis of this phenomenon.

With a single definition for *each*, I have presented a *unified* analysis of several different *each* constructions: the extension from transitive to ditransitive cases seems relatively mechanical, but finding a way to make the same denotation work also in noun-modifying prepositional phrases was a major accomplishment.

Finally, as part of the semantic framework, I discard the idea that singular and plural noun phrases are of different types. While this idea is not new, I have made the novel observation that existence of dual number makes it unnecessary and unuseful to maintain different underlying types for singular and plural.

Appendix A

Quantifier properties

It has been shown [2, 5] that generalised quantifiers, and the determiners that build them, behave according to a few universal rules. These rules are usually stated in terms of intersection between the noun argument of the determiner (of type *et*) and the verb argument of the quantifier (also of type *et*). Obviously, we will need to modify these somewhat in order to account for our new *ett* type for intransitive verbs.

First, we give the four rules as presented in Chapter 14 of [9], along with some examples. In the examples, we will assume an initial universe E consisting of Mike, Matt, Tim, and Michelle: Mike, Matt, and Tim are men (Michelle is not), and Mike, Matt, and Michelle walk (Tim does not). The four properties are true of all natural language determiners D, but we will present examples of pseudo-determiners that violate each rule (or rather, would violate the rule if they were determiners)—and that furthermore violate *only one* rule; this should demonstrate that no rules is redundant with another. In the formal definitions, the variables D and E are used as described above, and A and B represent the predicates over which D operates (in the examples, these will be $[\![\text{man}]\!]$ and $[\![\text{walk}]\!]$, respectively).

Conservativity If $A, B \subseteq E$ then $D_E(A)(B) \equiv D_E(A)(A \cap B)$.

This property of determiners states that we only need to consider those elements of B that are also in A when determining the model-theoretic truth value of the expression. With A as $[\![\text{man}]\!]$ and B as $[\![\text{walk}]\!]$, we can paraphrase as "If it is true that D men walk, then it is also true that D men are men who walk; furthermore, if it is true that D men are men who walk, then it is also true that D men walk." If we substitute for D various quantifiers, such as *all* or *some* or *no*, we see that this property does indeed hold.

An example of a non-conservative determiner would be *only*, if *only* were a determiner.[1] We can see that if D were a determiner meaning *only*, the second half of the sentence would not

[1] It isn't. Note that phrases like "only the men ..." are perfectly good; if *only* were a determiner it could not precede *the*.

be true: "only men are men who walk" is tautologically true, but "only men walk" is false in the example case, where Michelle walks too.

Extension (Constancy) If $A, B \subseteq E \subseteq E'$ then $D_E(A)(B) \equiv D_{E'}(A)(B)$.

The property of Extension tells us that the only things that matter in calculating the truth value of DAB are those entities in the union of A and B—the minimal possible E that satisfies the precondition is $A \cup B$. Anything else could fall into E', and therefore (according to this property), doesn't matter.

We could imagine a determiner *snork* that means something like "there are more A that B than there are non-A entities that don't B." More formally, $|A \cap B| > |\overline{A} \cap \overline{B}|$. Then the proposition "Snork men walk" would be true in our universe with just four entities (of which two are men who walk and none are non-men who don't walk). The same proposition would not be true if we added at least two non-walking women to that universe, or for that matter at least two (non-walking) aliens or chalkboards or anything else. Thus since Extension is a universal of natural language determiners, *snork* could not be a real determiner.

Quantity (Isomorphism) If F is a bijection from M_1 to M_2, then $D_{E_1}(A)(B) \equiv D_{E_2}(F(A))(F(B))$.

This property is what says that a determiner always builds a *quantifier*, as such, meaning that the specific contents of sets don't matter, just their cardinalities. A 'bijection' is a mapping that a mathematician would say is both 'one to one' and 'onto'; that is, in a bijection, every element of the domain maps onto a distinct element of the range, and every element of the range is mapped onto by a distinct element of the domain. One type of bijection, which we will use in our example, is where the mapping simply permutes the input set.

To demonstrate how this would work with a real determiner, consider the proposition "Every woman walks" in our example universe. We know it to be true because the set of women

$$[\![\text{woman}]\!] = \{\text{Michelle}\}$$

is a subset of the set of walkers

$$[\![\text{walk}]\!] = \{\text{Mike}, \text{Matt}, \text{Michelle}\}\,.$$

Now imagine that we apply the mapping function

$$F = \{\text{Mike} \rightarrow \text{Matt}, \text{Matt} \rightarrow \text{Tim}, \text{Tim} \rightarrow \text{Michelle}, \text{Michelle} \rightarrow \text{Mike}\}$$

to each of them.[2]

$$F([\![\text{woman}]\!]) = \{\text{Mike}\}$$
$$F([\![\text{walk}]\!]) = \{\text{Matt}, \text{Tim}, \text{Mike}\}$$

[2] The astute reader will note some handwaving as to the type of F—as presented, it is a function from entities to entities, but then it is used as a function from *sets* of entities to *sets* of entities. This notation is carried over from [9] and is discussed in more detail on page 53.

It is important to avoid falling into the cognitive trap of seeing $F([\![\text{woman}]\!])$ and thinking its value has anything obvious to do with the value of $[\![\text{woman}]\!]$; we need only look at the fact that $F([\![\text{woman}]\!])$ is a subset of $F([\![\text{walk}]\!])$ to know that $[\![\text{every}]\!](F([\![\text{woman}]\!]))(F([\![\text{walk}]\!]))$ is true just as $[\![\text{every}]\!]([\![\text{woman}]\!])([\![\text{walk}]\!])$ is.

Now consider the imaginary determiner *meef*, which means "all members of A whose names start with M do B". Then in our example universe, the proposition "Meef men walk" would be true. However, this definition is sensitive to the actual identity (in this case, the name) of the members in the set. If we apply the mapping function F, then we get the following new values (old values provided for comparison):

	P	$F(P)$
$[\![\text{man}]\!]$	{Mike, Matt, Tim}	{Matt, Tim, Michelle}
$[\![\text{walk}]\!]$	{Mike, Matt, Michelle}	{Matt, Tim, Mike}

Note that it's not true that all members of $F([\![\text{man}]\!])$ whose names start with M do $F([\![\text{walk}]\!])$, since Michelle is in $F([\![\text{man}]\!])$ but not $F([\![\text{walk}]\!])$. Thus, while $[\![\text{meef}]\!]([\![\text{man}]\!])([\![\text{walk}]\!])$ is true, $[\![\text{meef}]\!](F([\![\text{man}]\!]))(F([\![\text{walk}]\!]))$ is not; therefore *meef* does not have the Quantity property, and could not be a real natural language determiner.

Variation For each domain E there is a domain E' such that $A, B, C, E \subseteq E'$, such that $D_{E'}(A)(B)$ and $\neg D_{E'}(A)(C)$.

This property essentially tells us that determiners are nontrivial; for any given universe of entities E there is at least some superset thereof that lets us find some predicate B for which the quantifier $D(A)$ holds, and some other predicate C for which the quantifier does not hold.

Imagine a determiner *fromk*, which relates two predicates A and B if there exist any A in the universe (i.e. ignoring the value of the predicate B). "Fromk men walk" is true, because Mike and Matt and Tim—men—are all in our initial universe. Adding entities to the universe will not change the fact that there are men in the universe, so there is no predicate C to make this proposition false. Therefore this determiner does not meet the Variation criterion, and could not be a real determiner.

Collectively, these four properties make up a universal that might be expressed roughly as follows: the truth value of a determiner with arguments A and B is only affected by those entities that are members of A—neither by those entities in B but not A[3], nor by those entities in neither A nor B.[4] The only property of an entity that may be referenced is whether it is also a member of B (or not).[5] This property must, however, be referenced in some way.[6] Put yet another way, every natural

[3] Conservativity

[4] Extension

[5] Quantity

[6] Variation

language determiner is a predication on the cardinality of the set $A \cap B$, specified either absolutely or relative to the cardinality of A itself.

The order that the properties are presented here is not accidental; in a sense, it proceeds from profundity towards obviousness. That is, for each new property it becomes increasingly difficult to find a definition that feels to us like it *could* be a determiner if not for the rule. Conservativity is a profound result; something like 'only' seems like it's really close to being a determiner—why *aren't* there any determiners like that? At the other end of the scale we have 'fromk', a nearly meaningless pseudodeterminer that just goes to show that the Variation property is merely included for completeness' sake.

For later reference, here are some formally-written definitions for three real determiners plus the four fake determiners given above:

$$[\![\text{every}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\forall x_e [x \in P \rightarrow x \in Q]] \tag{A.1}$$

$$[\![\text{some}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\exists x_e [x \in P \wedge x \in Q]] \tag{A.2}$$

$$[\![\text{no}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\neg\exists x_e [x \in P \wedge x \in Q]] \tag{A.3}$$

$$[\![\text{only}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\forall x_e [x \in Q \rightarrow x \in P]] \tag{A.4}$$

$$[\![\text{snork}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} \left[|P \cap Q| > |\overline{P} \cap \overline{Q}|\right] \tag{A.5}$$

$$[\![\text{meef}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\{x_e : x \in P \wedge \text{name-starts-with-M}(x)\} \subseteq Q] \tag{A.6}$$

$$[\![\text{fromk}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}} [\exists x_e [P(x)]] \tag{A.7}$$

Now, we need to consider how to modify these properties to live in a universe where B is of type *ett* instead of *et*. Our examples will remain the same, except that [[walk]], being fully distributive, will now contain the singleton sets containing Mike, Matt, and Michelle as well as every combination thereof. The definitions of our determiners (and pseudodeterminers) will change to the following (prose descriptions of the pseudodeterminers appears below):

$$[\![\text{every}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} [\forall x_e [x \in P \rightarrow \{x\} \in \mathrm{Q}]] \tag{A.8}$$

$$[\![\text{some}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} [\exists \mathrm{X}_{et} [\mathrm{X} \subseteq P \wedge \mathrm{X} \in \mathrm{Q}]] \tag{A.9}$$

$$[\![\text{no}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} [\neg\exists \mathrm{X}_{et} [\mathrm{X} \subseteq P \wedge \mathrm{X} \in \mathrm{Q}]] \tag{A.10}$$

$$[\![\text{only}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} [\forall \mathrm{X}_{et} [X \in \mathrm{Q} \rightarrow \mathrm{X} \subseteq P]] \tag{A.11}$$

$$\begin{aligned} [\![\text{snork}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} & [\exists \mathrm{X}_{et} [\mathrm{X} \subseteq P \wedge \mathrm{X} \in \mathrm{Q} \\ & \wedge \forall \mathrm{Y}_{et} \left[\mathrm{Y} \subseteq \overline{P} \wedge \mathrm{Y} \neg \in \mathrm{Q} \rightarrow |X| > |Y|\right]]] \end{aligned} \tag{A.12}$$

$$[\![\text{meef}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{Q}} [\{x_e : x \in P \wedge \text{name-starts-with-M}(x)\} \in Q] \tag{A.13}$$

$$[\![\text{fromk}]\!] = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{ett}{Q}} [\exists x_e[P(x)]] \tag{A.14}$$

Now we are prepared to discuss how the formal definitions of each property needs to change in order to preserve the basic idea.

We will consider Variation first, as it is the most easily handled. The import of this condition will be to force the existence, at least in some larger universe, of at least one predicate where the quantifier is true and one where it is false. In fact, the main text of the condition need not change, just the types of some of the elements. Specifically, we'll be changing B and C to be subsets of $\mathcal{P}(E)$: For each domain E there is a domain E' such that $A, E \subseteq E'$ and $B, C \subseteq \mathcal{P}(E')$, such that $D_{E'}(A)(B)$ and $\neg D_{E'}(A)(C)$. Since our pseudodeterminer counterexample (*fromk*) doesn't even consider the value of B, it should be unsurprising that it remains a counterexample to the new version of the property; "Fromk men walk" is true in our example universe, and will remain true no matter what we add to the universe—no C will make it false.

Skipping around a little bit, we next consider the Extension property, which tells us that DAB is unaffected by those entities that are in neither A nor in B. As in the previous case, the condition can remain unchanged, but the preconditions are modified according to the new types of the elements. In particular, B now needs to be a subset of $\mathcal{P}(E)$: If $A \subseteq E \subseteq E'$ and $B \subseteq \mathcal{P}(E)$ then $D_E(A)(B) \equiv D_{E'}(A)(B)$.

Now consider the case of our pseudodeterminer *snork*. We would now rephrase it as "there is some subset X of P that Qs, that is larger than *any* subset Y of $\overline{P}$ that does not Q." This is written out formally in equation A.12 above. Take a moment and convince yourself that this is the logical equivalent to the definition given earlier in equation A.5.

Now, just as this pseudodeterminer was a counterexample under the old system, we find that it is a counterexample under the new system as well: "Snork men walk" is true in the example universe, but merely adding a couple of slithering aliens or chalkboards to it will make the proposition false, violating this property. This is because suddenly—in E'—there *does* exist a Y, namely the one containing aliens and blackboards, that is a subset of $\overline{P}$ (because there are no men in it), not in Q (because nothing in it walks), and yet still larger than the largest possible X (which is still just the set containing the two walking men).

Next we come to the Quantity property. In the original version, as stated, we have a mapping F that is a function from entities to entities, which is however applied to a *set* of entities, implicitly returning a set of entities. This is perhaps a slight abuse of the notation, but the meaning should be clear: operationally speaking, we are to take the output of this application to be the set consisting of the individual output of F when applied to each value in the input set. That is, a mapping.

If we may abuse the notation further, we could introduce the notion of a *deep* mapping, wherein the function F is applied to the atomic elements of its argument, whether that argument be a set or a set of sets. As such, the statement of the condition can remain unchanged: $D_{E_1}(A)(B) \equiv D_{E_2}(F(A))(F(B))$

An example should be illustrative of the deep-mapping concept, of this property, and of the behaviour of our counterexample. If you'll recall, our Quantity counterexample, *meef*, stated that "All members of A whose names start with M do B". As formalised in equation A.13, this might be better expressed as "The set containing exactly those members of A whose names begin with M is itself a member of B." As before, the statement "Meef men walk" is true, since Mike and Matt are the men with M names, and they both walk. Here, however, is what happens when we perform the mapping:

	P	$F(P)$
⟦man⟧	{Mike, Matt, Tim}	{Matt, Tim, Michelle}
⟦walk⟧	{ {Mike},	{ {Matt},
	{Matt},	{Tim},
	{Michelle},	{Mike},
	{Mike, Matt},	{Matt, Tim},
	{Mike, Michelle},	{Matt, Mike},
	{Matt, Michelle},	{Tim, Mike},
	{Mike, Matt, Michelle} }	{Matt, Tim, Mike} }

Reading off the right-hand column, we see that the set containing members of F(⟦man⟧) whose names start with M is {Matt, Michelle}, which is *not* a member of F(⟦walk⟧). Since ⟦meef⟧(⟦man⟧)(⟦walk⟧) is true but ⟦meef⟧(F(⟦man⟧))(F(⟦walk⟧)) is false, *meef* still does not have the Quantity property.

Finally, we come to the property of Conservativity. To extend the concept of conservativity to our *ett* universe, we simply replace "$A \cap B$" with "$\mathcal{P}(A) \cap B$". This extracts from B all those sets comprised solely of members of A. A moment's reflection will reveal that this (like the map solution in the previous paragraph) reduces in the distributive case to the *et* definition. But this definition will be true of the non-distributive predicates as well; a formal proof will be omitted here, but for an intuition into the reasoning, consider that this new conservativity definition is equivalent to one saying "a determiner is conservative if, in its definition, '$\mathrm{X} \in \mathrm{Q}$' can be replaced with with '$\mathrm{X} \in \mathrm{Q} \wedge \mathrm{X} \subseteq P$', and vice versa." In many cases, this extra term is already present in the definition, and in the rest (of the true determiners) it is easy to see that it is tautological.

Only is not conservative, under this definition. Returning to our running example: "Only men walk." The value of $\mathcal{P}$(⟦man⟧) $\cap$ ⟦walk⟧ is {{Mike}, {Matt}, {Mike, Matt}}. Since each member of that set of sets is a subset of ⟦man⟧, the right-hand part of the condition is true: ⟦only⟧(⟦man⟧)($\mathcal{P}$(⟦man⟧) $\cap$ ⟦walk⟧). However, the left-hand side is not true, since (among others) the set {Michelle} is in ⟦walk⟧, and is not a subset of ⟦man⟧. Thus the equivalence does not hold, and *only* does not have the Conservative property.

So, to sum up, our new quantifier universals:

Conservativity If $A \subseteq E$ and $B \subseteq \mathcal{P}(E)$ then $D_E(A)(B) \equiv D_E(A)(\mathcal{P}(A) \cap B)$.

Extension (Constancy) If $A \subseteq E \subseteq E'$ and $B \subseteq \mathcal{P}(E)$ then $D_E(A)(B) \equiv D_{E'}(A)(B)$.

Quantity (Isomorphism) If F is a bijection from M_1 to M_2, then $D_{E_1}(A)(B) \equiv D_{E_2}(F(A))(F(B))$.

Variation For each domain E there is a domain E' such that $A, E \subseteq E'$ and $B, C \subseteq \mathcal{P}(E')$, such that $D_{E'}(A)(B)$ and $\neg D_{E'}(A)(C)$.

They still *mean* the same as they did, in the sense that their import is unchanged. They can also be summarised in a very similar way: The truth value of a determiner with arguments A and B is only affected by those entities that are members of A—neither by those elements of B containing entities not in A[7], nor by those sets *not* in B containing entities not in A.[8] The only property of an entity that may be referenced is whether it is also in a member of B (or not).[9] This property must, however, be referenced in some way.[10] Put yet another way, every natural language determiner is a predication on the existence of a member of the set $\mathcal{P}(A) \cap B$ with a specific cardinality, specified either absolutely or relative to the cardinality of A itself.

Since we *can* actually define all the real determiners in terms of intersection and cardinality, now would be a good time to do so. First under the old *et* framework:

$$
\begin{array}{rll}
[\![\text{every}]\!] & = \lambda \overset{CN}{\underset{et}{P}} \lambda \overset{vb}{\underset{et}{Q}}[& |P \cap Q| = |P| \quad] \\
[\![\text{some}]\!] & & |P \cap Q| \geq 1 \\
[\![\text{no}]\!] & & |P \cap Q| = 0 \\
[\![\text{most}]\!] & & |P \cap Q| \geq \frac{1}{2}|P| \\
[\![\text{several}]\!] & & |P \cap Q| \geq\sim 5 \\
[\![\text{few}]\!] & & |P \cap Q| \leq\sim 4
\end{array}
$$

Now, the same determiners in the *ett* framework:

$$
\begin{array}{rlll}
[\![\text{every}]\!] & = \lambda \overset{CN}{\underset{et}{P}} \; \lambda \overset{vb}{\underset{ett}{\mathrm{Q}}} \; [& \exists \mathrm{X}_{et}[\mathrm{X} \in (\mathcal{P}(P) \cap \mathrm{Q}) \wedge & |\mathrm{X}| \geq |P| \quad] \\
[\![\text{some}]\!] & & & |\mathrm{X}| \geq 1 \\
[\![\text{no}]\!] & & \neg & |\mathrm{X}| \geq 1 \\
[\![\text{most}]\!] & & & |\mathrm{X}| \geq \tfrac{1}{2}|P| \\
[\![\text{several}]\!] & & & |\mathrm{X}| \geq\sim 5 \\
[\![\text{few}]\!] & & \neg & |\mathrm{X}| \geq\sim 4
\end{array}
$$

Interestingly, those determiners that make predications on the *non*-existence of a set of the appropriate cardinality seem to be exactly those which license negative-polarity items. This requires further investigation.

Also interesting is that in this framework, all the cardinality requirements can be expressed in terms of $\geq$. This is probably not deeply profound, but it is at least a mildly interesting coincidence.

[7] Conservativity

[8] Extension

[9] Quantity

[10] Variation

Appendix B

Derivations

Here we present more detailed derivations for some of the definitions given in the main body of the thesis. In some cases, for readability the value reported in the main body of the thesis is not in its most reduced form; the derivation given below continues past this point, and the symbol ☞ will be used to indicate the version given in the text.

Chapter 4

$$
\begin{array}{rcl}
\multicolumn{3}{l}{[\![\text{The kids read three books each.}]\!]:} \\
\text{“three…each”:DP} & t & \forall z[(z \in Z_i) \to \exists X[\text{3books}'(X) \wedge R_j(z, X)]] \\
\text{“bought”:V} & EEt & \text{bought}' \\
\text{“bought…each”:VP} & t & \forall z[(z \in Z_i) \to \exists X[\text{3books}'(X) \wedge \text{bought}'(z, X)]] \\
 & & \quad \text{(index-triggered } \lambda\text{-abstraction, functional application)} \\
\text{“the…each”:IP} & t & \forall z[(z \in [\![\text{the kids}]\!]) \to \exists X[\text{3books}'(X) \wedge \text{bought}'(z, X)]] \\
 & & \quad \text{(index-triggered } \lambda\text{-abstraction, functional application)}
\end{array} \tag{4.9}
$$

$$
\begin{array}{rcl}
\multicolumn{3}{l}{[\![\text{gloves with four fingers each}]\!]:} \\
\text{“four…each”:DP} & t & \forall z[(z \in Z_i) \to \exists X[\text{4fingers}'(X) \wedge R_j(z, X)]] \\
\text{“with”:P} & EEt & \text{with}' \\
\text{“with…each”:PP} & t & \forall z[(z \in Z_i) \to \exists X[\text{4fingers}'(X) \wedge \text{with}'(z, X)]] \\
 & & \quad \text{(index-triggered } \lambda\text{-abstraction, functional application)} \\
\text{“gloves…each”:NP} & Et & \lambda Z[\text{gloves}'(Z) \wedge \forall z[(z \in Z) \to \exists X[\text{4fingers}'(X) \wedge \text{with}'(z, X)]] \\
 & & \quad \text{(index-triggered } \lambda\text{-abstraction, predicate modification)}
\end{array} \tag{4.10}
$$

Chapter 5

$$
\begin{aligned}
[\![\text{some stories}]\!]_{ettt} &= [\![\text{some}]\!]([\![\text{stories}]\!]) \\
&= \lambda \underset{et}{\overset{noun}{P}} \left[\lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq P \wedge \mathrm{Q}(\mathrm{X})]] \right] (\{o, h, k\}) \\
&= \lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq \{o, h, k\} \wedge \mathrm{Q}(\mathrm{X})]]
\end{aligned}
\tag{5.18}
$$

$$
\begin{aligned}
[\![\text{no kids}]\!]_{ettt} &= [\![\text{no}]\!]([\![\text{kids}]\!]) \\
&= \lambda \underset{et}{\overset{noun}{P}} \left[\lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\neg\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq P \wedge \mathrm{Q}(\mathrm{X})]] \right] (\{a, b, c\}) \\
&= \lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\neg\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq \{a, b, c\} \wedge \mathrm{Q}(\mathrm{X})]]
\end{aligned}
\tag{5.19}
$$

$$
\begin{aligned}
[\![\text{no kids sleep}]\!]_{t} &= [\![\text{no kids}]\!]([\![\text{sleep}]\!]) \\
&= \lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\neg\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq \{a, b, c\} \wedge \mathrm{Q}(\mathrm{X})]] \, ([\![\text{sleep}]\!]) \\
&= \neg\exists \mathrm{X}_{et} \, [\mathrm{X} \subseteq \{a, b, c\} \wedge [\![\text{sleep}]\!](\mathrm{X})]
\end{aligned}
\tag{5.20}
$$

$$
\begin{aligned}
&[\![\text{read some stories}]\!]_{ett} \\
&\quad = \text{arglift}([\![\text{read}]\!])([\![\text{some stories}]\!]) \\
&\quad = \lambda \underset{ettt}{\overset{obj}{\mathscr{P}}} \left[\lambda \underset{et}{\overset{sbj}{\mathrm{X}}} \left[\mathscr{P} \left(\lambda \underset{et}{\overset{obj}{\mathrm{Y}}} \, [[\![\text{read}]\!](Y)(X)] \right) \right] \right] \left(\lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\exists \mathrm{Z} \, [\mathrm{Z} \subseteq \{o, h, k\} \wedge \mathrm{Q}(\mathrm{Z})]] \right) \\
&\quad = \lambda \underset{et}{\overset{sbj}{\mathrm{X}}} \left[\lambda \underset{ett}{\overset{vb}{\mathrm{Q}}} \, [\exists \mathrm{Z} \, [\mathrm{Z} \subseteq \{o, h, k\} \wedge \mathrm{Q}(\mathrm{Z})]] \left(\lambda \underset{et}{\overset{obj}{\mathrm{Y}}} \, [[\![\text{read}]\!](Y)(X)] \right) \right] \\
&\quad = \lambda \underset{et}{\overset{sbj}{\mathrm{X}}} \left[\exists \mathrm{Z} \left[\mathrm{Z} \subseteq \{o, h, k\} \wedge \lambda \underset{et}{\overset{obj}{\mathrm{Y}}} \, [[\![\text{read}]\!](Y)(X)] \, (\mathrm{Z}) \right] \right] \\
&\quad = \lambda \underset{et}{\overset{sbj}{\mathrm{X}}} \, [\exists \mathrm{Z} \, [\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(X)]]
\end{aligned}
\tag{5.23}
$$

$$
\begin{aligned}
&[\![\text{Chris read some stories}]\!]_{t} \\
&\quad = [\![\text{read some stories}]\!]([\![\text{Chris}]\!]) \\
&\quad = \lambda \underset{et}{\overset{sbj}{\mathrm{X}}} \, [\exists \mathrm{Z} \, [\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(X)]] \, (\{c\}) \\
&\quad = \exists \mathrm{Z} \, [\mathrm{Z} \subseteq \{o, h, k\} \wedge [\![\text{read}]\!](Z)(\{c\})]
\end{aligned}
\tag{5.24}
$$

$$
\begin{aligned}
&[\![\text{read no stories}]\!]_{ett}\\
&\quad= \text{arglift}([\![\text{read}]\!])([\![\text{no stories}]\!])\\
&\quad= \overset{obj}{\underset{ettt}{\lambda\mathscr{P}}}\left[\overset{sbj}{\underset{et}{\lambda \mathrm{X}}}\left[\mathscr{P}\left(\overset{obj}{\underset{et}{\lambda \mathrm{Y}}}\,[[\![\text{read}]\!](Y)(X)]\right)\right]\right]\left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\,[\neg\exists \mathrm{Z}\,[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, \mathrm{Q(Z)}]]\right)\\
&\quad= \overset{sbj}{\underset{et}{\lambda \mathrm{X}}}\left[\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\,[\neg\exists \mathrm{Z}\,[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, \mathrm{Q(Z)}]]\left(\overset{obj}{\underset{et}{\lambda \mathrm{Y}}}\,[[\![\text{read}]\!](Y)(X)]\right)\right]\\
&\quad= \overset{sbj}{\underset{et}{\lambda \mathrm{X}}}\left[\neg\exists \mathrm{Z}\left[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, \overset{obj}{\underset{et}{\lambda \mathrm{Y}}}\,[[\![\text{read}]\!](Y)(X)]\,(\mathrm{Z})\right]\right]\\
&\quad= \overset{sbj}{\underset{et}{\lambda \mathrm{X}}}\,[\neg\exists \mathrm{Z}\,[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, [\![\text{read}]\!](Z)(X)]]
\end{aligned}
\tag{5.25}
$$

$$
\begin{aligned}
&[\![\text{Bill read no stories}]\!]_{t}\\
&\quad= [\![\text{read no stories}]\!]([\![\text{Bill}]\!])\\
&\quad= \overset{sbj}{\underset{et}{\lambda \mathrm{X}}}\,[\neg\exists \mathrm{Z}\,[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, [\![\text{read}]\!](Z)(X)]]\,(\{b\})\\
&\quad= \neg\exists \mathrm{Z}\,[\mathrm{Z} \subseteq \{o,h,k\} \,\wedge\, [\![\text{read}]\!](Z)(\{b\})]
\end{aligned}
\tag{5.26}
$$

$$
\begin{aligned}
&[\![\text{Some boy sleeps}]\!]\\
&\quad= [\![\text{some boy}]\!]([\![\text{sleep+SG}]\!])\\
&\quad= \overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, \mathrm{Q(X)}]\right](\overset{sbj}{\underset{et}{\lambda \mathrm{Z}}}\,[[\![\text{sleep}]\!](\mathrm{Z}) \,\wedge\, |\mathrm{Z}| = 1])\\
&\quad= \underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, [\![\text{sleep}]\!](\mathrm{X}) \,\wedge\, |\mathrm{X}| = 1]
\end{aligned}
\tag{5.29}
$$

$$
\begin{aligned}
&[\![\text{Some boys sleep}]\!]\\
&\quad= [\![\text{some boy}]\!]([\![\text{sleep+PL}]\!])\\
&\quad= \overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, \mathrm{Q(X)}]\right](\overset{sbj}{\underset{et}{\lambda \mathrm{Z}}}\,[[\![\text{sleep}]\!](\mathrm{Z}) \,\wedge\, |\mathrm{Z}| > 1])\\
&\quad= \underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, [\![\text{sleep}]\!](\mathrm{X}) \,\wedge\, |\mathrm{X}| > 1]
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
&[\![\text{No boy sleeps.}]\!]\\
&\quad= [\![\text{no boy}]\!]([\![\text{sleep+SG}]\!])\\
&\quad= \overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\neg\underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, \mathrm{Q(X)}]\right](\overset{sbj}{\underset{et}{\lambda \mathrm{Z}}}\,[[\![\text{sleep}]\!](\mathrm{Z}) \,\wedge\, |\mathrm{Z}| = 1])\\
&\quad= \neg\underset{et}{\exists \mathrm{X}}\,[\mathrm{X} \subseteq \{a,b,c\} \,\wedge\, [\![\text{sleep}]\!](\mathrm{X}) \,\wedge\, |\mathrm{X}| = 1]
\end{aligned}
\tag{5.31}
$$

Chapter 6

$$\begin{aligned}
&\llbracket \underset{\langle etett,\ ett\rangle}{\text{two books each}} \rrbracket\\
&= \llbracket \text{each} \rrbracket\,(\llbracket \text{two books} \rrbracket)\\
&= \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right]\right](\llbracket \text{two books} \rrbracket)\\
&= \overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\llbracket \text{two books} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right]\\
&= \overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\underset{et}{\lambda \mathrm{Z}}\,[\mathrm{Z} \subseteq \llbracket \text{books} \rrbracket \wedge |\mathrm{Z}| = 2]\,(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})\right]\right]\right]\right]\\
&= \overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{Y} \subseteq \llbracket \text{books} \rrbracket \wedge |\mathrm{Y}| = 2 \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right]\\
&\llbracket \underset{ett}{\text{read two books each}} \rrbracket\\
&= \llbracket \text{two books each} \rrbracket(\llbracket \text{read} \rrbracket)\\
&= \overset{rel}{\underset{etett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{Y} \subseteq \llbracket \text{books} \rrbracket \wedge |\mathrm{Y}| = 2 \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right](\llbracket \text{read} \rrbracket)\\
&= \overset{rng}{\underset{et}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{Y} \subseteq \llbracket \text{books} \rrbracket \wedge |\mathrm{Y}| = 2 \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{x\})]\right]\right] \qquad (6.4)
\end{aligned}$$

$$\begin{aligned}
&\llbracket \text{read the kids one story each} \rrbracket\\
&= \llbracket \text{each} \rrbracket(\llbracket \text{one story} \rrbracket)(\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
&= \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{rel}{\underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\lambda \mathscr{R}}}\left[\underset{\vec{b}}{\lambda \vec{B}}\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\underset{\vec{a}}{\lambda \vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\right]\\
&\qquad (\llbracket \text{one story} \rrbracket)(\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
&= \overset{rel}{\underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\lambda \mathscr{R}}}\left[\underset{\vec{b}}{\lambda \vec{B}}\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\underset{\vec{a}}{\lambda \vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\\
&\qquad (\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
&\qquad \textit{\dots treating } \vec{b} \textit{ as null, and } \vec{a} \textit{ as } \langle e,t\rangle \dots\\
&= \overset{rel}{\underset{etetett}{\lambda \mathscr{R}}}\left[\overset{rng}{\underset{et}{\lambda \mathrm{X}}}\underset{et}{\lambda \mathrm{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right]\right](\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
&= \overset{rng}{\underset{et}{\lambda \mathrm{X}}}\underset{et}{\lambda \mathrm{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\,[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right](\llbracket \text{the kids} \rrbracket)\\
&\text{☞}\ \underset{et}{\lambda \mathrm{A}}\left[\forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\,[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right]\\
&= \underset{et}{\lambda \mathrm{A}}\left[\forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\left[\underset{et}{\lambda \mathrm{Z}}\,[\mathrm{Z} \subseteq \llbracket \text{story} \rrbracket \wedge |\mathrm{Z}| = 1]\,(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})\right]\right]\right]\\
&= \underset{et}{\lambda \mathrm{A}}\left[\forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\,[\mathrm{Y} \subseteq \llbracket \text{story} \rrbracket \wedge |\mathrm{Y}| = 1 \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right] \qquad (6.10)
\end{aligned}$$

$$
\begin{aligned}
&\llbracket \text{The kids read Chris one story each} \rrbracket\\
=\ & \llbracket \text{each} \rrbracket(\llbracket \text{one story} \rrbracket)(\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
=\ & \overset{dist}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{rel}{\underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\lambda \mathscr{R}}}\left[\lambda \underset{b}{\vec{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\lambda \underset{\vec{a}}{\vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\right]\\
& \quad (\llbracket \text{one story} \rrbracket)(\llbracket \text{read} \rrbracket)(\llbracket \text{Chris} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
=\ & \overset{rel}{\underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\lambda \mathscr{R}}}\left[\lambda \underset{b}{\vec{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\lambda \underset{\vec{a}}{\vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\\
& \quad (\llbracket \text{read} \rrbracket)(\llbracket \text{Chris} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
& \ldots \textit{treating } \vec{b} \textit{ as } \langle e,t\rangle, \textit{ and } \vec{a} \textit{ as null}\ldots\\
=\ & \overset{rel}{\underset{etetett}{\lambda \mathscr{R}}}\left[\lambda \underset{et}{\mathrm{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\mathrm{B})(\{x\})\right]\right]\right]\right]\\
& \quad (\llbracket \text{read} \rrbracket)(\llbracket \text{Chris} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
=\ & \lambda \underset{et}{\mathrm{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\mathrm{B})(\{x\})\right]\right]\right] (\llbracket \text{Chris} \rrbracket)(\llbracket \text{the kids} \rrbracket)\\
=\ & \lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{c\})(\{x\})\right]\right]\right] (\llbracket \text{the kids} \rrbracket)\\
☞\ & \forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{c\})(\{x\})\right]\right]\\
=\ & \forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\left[\lambda \underset{et}{\mathrm{Z}}\left[\mathrm{Z} \subseteq \llbracket \text{story} \rrbracket \wedge |\mathrm{Z}| = 1\right](\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{c\})(\{x\})\right]\right]\\
=\ & \forall \underset{e}{x} \in \llbracket \text{the kids} \rrbracket\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{Y} \subseteq \llbracket \text{story} \rrbracket \wedge |\mathrm{Y}| = 1 \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\{c\})(\{x\})\right]\right]
\end{aligned}
\tag{6.12}
$$

$$
\begin{aligned}
&\llbracket \text{Alex and Loren read the kids one story each.} \rrbracket\\
=\ & \llbracket \text{one story each} \rrbracket(\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)(\llbracket \text{Alex and Loren} \rrbracket)\\
=\ & \overset{rel}{\underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\lambda \mathscr{R}}}\left[\lambda \underset{b}{\vec{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\lambda \underset{\vec{a}}{\vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\\
& \quad (\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)(\{a,l\})\\
& \ldots \textit{treating } \vec{b} \textit{ as } \langle e,t\rangle, \textit{ and } \vec{a} \textit{ as null}\ldots\\
=\ & \overset{rel}{\underset{etetett}{\lambda \mathscr{R}}}\left[\lambda \underset{et}{\mathrm{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\mathrm{B})(\{x\})\right]\right]\right]\right]\\
& \quad (\llbracket \text{read} \rrbracket)(\llbracket \text{the kids} \rrbracket)(\{a,l\})\\
=\ & \lambda \underset{et}{\mathrm{B}}\lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\mathrm{B})(\{x\})\right]\right]\right] (\llbracket \text{the kids} \rrbracket)(\{a,l\})\\
=\ & \lambda \overset{rng}{\underset{et}{\mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\llbracket \text{the kids} \rrbracket)(\{x\})\right]\right]\right] (\{a,l\})\\
☞\ & \forall \underset{e}{x} \in \{a,l\}\left[\exists \underset{et}{\mathrm{Y}}\left[\llbracket \text{one story} \rrbracket(\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\llbracket \text{the kids} \rrbracket)(\{x\})\right]\right]\\
=\ & \forall \underset{e}{x} \in \{a,l\}\left[\exists \underset{et}{\mathrm{Y}}\left[\lambda \underset{et}{\mathrm{Z}}\left[\mathrm{Z} \subseteq \llbracket \text{story} \rrbracket \wedge |\mathrm{Z}| = 1\right](\mathrm{Y}) \wedge \llbracket \text{read} \rrbracket(\mathrm{Y})(\llbracket \text{the kids} \rrbracket)(\{x\})\right]\right]
\end{aligned}
$$

$$= \quad \forall \underset{e}{x} \in \{a,l\} \left[\exists \underset{et}{\mathrm{Y}} [\mathrm{Y} \subseteq [\![\text{story}]\!] \wedge |\mathrm{Y}| = 1 \wedge [\![\text{read}]\!](\mathrm{Y})([\![\text{the kids}]\!])(\{x\})]\right] \tag{6.14}$$

[\![Alex and Loren read the kids one story each.]\!]

$$= \quad [\![\text{one story each}]\!]([\![\text{read}]\!])([\![\text{the kids}]\!])([\![\text{Alex and Loren}]\!])$$

$$= \quad \underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}} \left[\overset{rng}{\lambda \underset{b}{\vec{B}} \lambda \underset{et}{\mathrm{X}} \lambda \underset{\vec{a}}{\vec{A}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} \left[[\![\text{one story}]\!](\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]$$

$$([\![\text{read}]\!])([\![\text{the kids}]\!])(\{a,l\})$$

. . . treating $\vec{b}$ as null, and $\vec{a}$ as $\langle e, t\rangle$. . .

$$= \quad \underset{etetett}{\overset{rel}{\lambda \mathscr{R}}} \left[\overset{rng}{\lambda \underset{et}{\mathrm{X}} \lambda \underset{et}{\mathrm{A}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{one story}]\!](\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right]\right]$$

$$([\![\text{read}]\!])([\![\text{the kids}]\!])(\{a,l\})$$

$$= \quad \overset{rng}{\lambda \underset{et}{\mathrm{X}} \lambda \underset{et}{\mathrm{A}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{one story}]\!](\mathrm{Y}) \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right] ([\![\text{the kids}]\!])(\{a,l\})$$

$$= \quad \lambda \underset{et}{\mathrm{A}} \left[\forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{one story}]\!](\mathrm{Y}) \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\})(\mathrm{A})]\right]\right] (\{a,l\})$$

$$☞ \quad \forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{one story}]\!](\mathrm{Y}) \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\})(\{a,l\})]\right]$$

$$= \quad \forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{\mathrm{Y}} \left[\lambda \underset{et}{\mathrm{Z}} [\mathrm{Z} \subseteq [\![\text{story}]\!] \wedge |\mathrm{Z}| = 1] (\mathrm{Y}) \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\})(\{a,l\})\right]\right]$$

$$= \quad \forall \underset{e}{x} \in [\![\text{the kids}]\!] \left[\exists \underset{et}{\mathrm{Y}} [\mathrm{Y} \subseteq [\![\text{story}]\!] \wedge |\mathrm{Y}| = 1 \wedge [\![\text{read}]\!](\mathrm{Y})(\{x\})(\{a,l\})]\right] \tag{6.15}$$

[\![bought three books each]\!]

$$= \quad [\![\text{each}]\!]([\![\text{three books}]\!])([\![\text{bought}]\!])$$

$$= \quad \underset{ett}{\overset{dist}{\lambda \mathrm{P}}} \left[\underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}} \left[\overset{rng}{\lambda \underset{b}{\vec{B}} \lambda \underset{et}{\mathrm{X}} \lambda \underset{\vec{a}}{\vec{A}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} \left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\right]$$

$$([\![\text{three books}]\!])([\![\text{bought}]\!])$$

$$= \quad \underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}} \left[\overset{rng}{\lambda \underset{b}{\vec{B}} \lambda \underset{et}{\mathrm{X}} \lambda \underset{\vec{a}}{\vec{A}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} \left[[\![\text{three books}]\!](\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right] ([\![\text{bought}]\!])$$

. . . so $\vec{b}$ and $\vec{a}$ are both null. . .

$$= \quad \underset{etett}{\overset{rel}{\lambda \mathscr{R}}} \left[\overset{rng}{\lambda \underset{et}{\mathrm{X}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{three books}]\!](\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})]\right]\right]\right] ([\![\text{bought}]\!])$$

$$= \quad \overset{rng}{\lambda \underset{et}{\mathrm{X}}} \left[\forall \underset{e}{x} \in \mathrm{X} \left[\exists \underset{et}{\mathrm{Y}} [[\![\text{three books}]\!](\mathrm{Y}) \wedge [\![\text{bought}]\!](\mathrm{Y})(\{x\})]\right]\right]$$

[\![some kids]\!]

$$= \quad \underset{et}{\overset{noun}{\lambda P}} \left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\exists \underset{et}{\mathrm{Z}} [\mathrm{Z} \subseteq P \wedge \mathrm{Q}(\mathrm{Z})]\right]\right] ([\![\text{kids}]\!])$$

$$= \quad \underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\exists \underset{et}{\mathrm{Z}} [\mathrm{Z} \subseteq [\![\text{kids}]\!] \wedge \mathrm{Q}(\mathrm{Z})]\right]$$

$$
\begin{aligned}
&\llbracket \text{Some kids bought three books each.} \rrbracket \\
&\quad = \llbracket \text{some kids} \rrbracket (\llbracket \text{bought three books each} \rrbracket) \\
&\quad = \lambda \underset{ett}{\overset{vb}{Q}} \left[\exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge Q(Z) \right] \right] \\
&\qquad\qquad \left(\lambda \underset{et}{\overset{rng}{X}} \left[\forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \right) \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge \lambda \underset{et}{\overset{rng}{X}} \left[\forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] (Z) \right] \\
&\quad \text{☞}\ \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\lambda \underset{et}{Z} \left[Z \subseteq \llbracket \text{books} \rrbracket \wedge |Z| = 3 \right] (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[Y \subseteq \llbracket \text{books} \rrbracket \wedge |Y| = 3 \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right]
\end{aligned}
\tag{6.16}
$$

$$
\begin{aligned}
&\llbracket \text{two kids} \rrbracket_{\text{GQ}} \\
&\quad = \text{numlift}(\llbracket \text{two kids} \rrbracket) \\
&\quad = \lambda \underset{ett}{\overset{NumP}{P}} \left[\lambda \underset{ett}{\overset{vb}{Q}} \left[\exists \underset{et}{Z} \left[P(Z) \wedge Q(Z) \right] \right] \right] (\llbracket \text{two kids} \rrbracket) \\
&\quad = \lambda \underset{ett}{\overset{vb}{Q}} \left[\exists \underset{et}{Z} \left[\llbracket \text{two kids} \rrbracket (Z) \wedge Q(Z) \right] \right] \\
&\llbracket \text{Two kids bought three books each.} \rrbracket \\
&\quad = \llbracket \text{two kids} \rrbracket_{\text{GQ}} (\llbracket \text{bought three books each} \rrbracket) \\
&\quad = \lambda \underset{ett}{\overset{vb}{Q}} \left[\exists \underset{et}{Z} \left[\llbracket \text{two kids} \rrbracket (Z) \wedge Q(Z) \right] \right] \\
&\qquad\qquad \left(\lambda \underset{et}{\overset{rng}{X}} \left[\forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \right) \\
&\quad = \exists \underset{et}{Z} \left[\llbracket \text{two kids} \rrbracket (Z) \wedge \lambda \underset{et}{\overset{rng}{X}} \left[\forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] (Z) \right] \\
&\quad \text{☞}\ \exists \underset{et}{Z} \left[\llbracket \text{two kids} \rrbracket (Z) \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[\lambda \underset{et}{X} \left[X \subseteq \llbracket \text{kids} \rrbracket \wedge |X| = 2 \right] (Z) \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge |Z| = 2 \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge |Z| = 2 \right. \\
&\qquad\qquad \left. \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[\lambda \underset{et}{X} \left[X \subseteq \llbracket \text{books} \rrbracket \wedge |X| = 3 \right] (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \\
&\quad = \exists \underset{et}{Z} \left[Z \subseteq \llbracket \text{kids} \rrbracket \wedge |Z| = 2 \wedge \forall \underset{e}{x} \in Z \left[\exists \underset{et}{Y} \left[Y \subseteq \llbracket \text{books} \rrbracket \wedge |Y| = 3 \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right]
\end{aligned}
\tag{6.17}
$$

⟦gave _ four presents each⟧

$$= \llbracket\text{four presents each}\rrbracket(\llbracket\text{gave}\rrbracket)$$

$$= \underset{\langle et,\langle \vec{b},\langle et,\langle \vec{a},t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda\mathscr{R}}}\left[\lambda\underset{b}{\vec{B}}\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{\vec{a}}{\vec{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right](\llbracket\text{gave}\rrbracket)$$

... treating $\vec{b}$ as null, and $\vec{a}$ as et ...

$$= \underset{etetett}{\overset{rel}{\lambda\mathscr{R}}}\left[\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{et}{\mathrm{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\mathscr{R}(\mathrm{Y})(\{x\})(\mathrm{A})\right]\right]\right]\right](\llbracket\text{gave}\rrbracket)$$

$$= \overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{et}{\mathrm{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})\right]\right]\right]$$

⟦gave some girls four presents each⟧

$$= \text{arglift}(\llbracket\text{gave \_ four presents each}\rrbracket)(\llbracket\text{some girls}\rrbracket)$$

$$= \underset{\langle et,\langle et,t\rangle\rangle}{\overset{verb}{\lambda\mathscr{R}}}\left[\underset{ettt}{\overset{obj}{\lambda\mathscr{P}}}\left[\underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\mathscr{P}\left(\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\mathscr{R}(\mathrm{W})(\mathrm{Z})\right]\right)\right]\right]\right]$$
$$\left(\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{et}{\mathrm{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})\right]\right]\right]\right)(\llbracket\text{some girls}\rrbracket)$$

$$= \underset{ettt}{\overset{obj}{\lambda\mathscr{P}}}\left[\underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\mathscr{P}\left(\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{et}{\mathrm{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\right.\right.\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\left.\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{A})\right]\right]\right](\mathrm{W})(\mathrm{Z})\right]\right)\right]\right](\llbracket\text{some girls}\rrbracket)$$

$$= \underset{ettt}{\overset{obj}{\lambda\mathscr{P}}}\left[\underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\mathscr{P}\left(\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\forall\underset{e}{x}\in\mathrm{W}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right]\right)\right]\right]$$
$$(\llbracket\text{some girls}\rrbracket)$$

$$= \underset{ettt}{\overset{obj}{\lambda\mathscr{P}}}\left[\underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\mathscr{P}\left(\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\forall\underset{e}{x}\in\mathrm{W}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right]\right)\right]\right]$$
$$\left(\underset{ett}{\overset{vb}{\lambda\mathrm{Q}}}\left[\exists\underset{et}{\mathrm{X}}\left[\mathrm{X}\subseteq\llbracket\text{girls}\rrbracket\wedge\mathrm{Q}(\mathrm{X})\right]\right]\right)$$

$$= \underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\underset{ett}{\overset{vb}{\lambda\mathrm{Q}}}\left[\exists\underset{et}{\mathrm{X}}\left[\mathrm{X}\subseteq\llbracket\text{girls}\rrbracket\wedge\mathrm{Q}(\mathrm{X})\right]\right]\right.$$
$$\left.\left(\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\forall\underset{e}{x}\in\mathrm{W}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right]\right)\right]$$

$$= \underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\exists\underset{et}{\mathrm{X}}\left[\mathrm{X}\subseteq\llbracket\text{girls}\rrbracket\right.\right.$$
$$\left.\left.\wedge\underset{et}{\overset{obj}{\lambda\mathrm{W}}}\left[\forall\underset{e}{x}\in\mathrm{W}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right](\mathrm{X})\right]\right]$$

$$= \underset{et}{\overset{sbj}{\lambda\mathrm{Z}}}\left[\exists\underset{et}{\mathrm{X}}\left[\mathrm{X}\subseteq\llbracket\text{girls}\rrbracket\wedge\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y})\wedge\llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right]\right]$$

$$
\begin{aligned}
&\llbracket\text{Alex gave some girls four presents each.}\rrbracket\\
&= \llbracket\text{gave some girls four presents each}\rrbracket(\llbracket\text{Alex}\rrbracket)\\
&= \overset{sbj}{\underset{et}{\lambda \mathrm{Z}}}\left[\underset{et}{\exists \mathrm{X}}\left[\mathrm{X} \subseteq \llbracket\text{girls}\rrbracket \wedge \underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\mathrm{Z})\right]\right]\right]\right](\{a\})\\
&☞\ \underset{et}{\exists \mathrm{X}}\left[\mathrm{X} \subseteq \llbracket\text{girls}\rrbracket \wedge \underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four presents}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\{a\})\right]\right]\right]\\
&= \underset{et}{\exists \mathrm{X}}\Big[\mathrm{X} \subseteq \llbracket\text{girls}\rrbracket\\
&\qquad \wedge \underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\underset{et}{\lambda \mathrm{Z}}\left[\mathrm{Z} \subseteq \llbracket\text{presents}\rrbracket \wedge |\mathrm{Z}| = 4\right](\mathrm{Y}) \wedge \llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\{a\})\right]\right]\Big]\\
&= \underset{et}{\exists \mathrm{X}}\left[\mathrm{X} \subseteq \llbracket\text{girls}\rrbracket \wedge \underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\mathrm{Y} \subseteq \llbracket\text{presents}\rrbracket \wedge |\mathrm{Y}| = 4 \wedge \llbracket\text{gave}\rrbracket(\mathrm{Y})(\{x\})(\{a\})\right]\right]\right]
\end{aligned}
\tag{6.18}
$$

$$
\begin{aligned}
&\llbracket\text{with four fingers}\rrbracket(\text{bad; using old } \textit{with} \text{ definition})\\
&= \llbracket\text{with}\rrbracket(\llbracket\text{four fingers}\rrbracket)\\
&= \llbracket\text{with}\rrbracket(\text{numlift}(\llbracket\text{four fingers}\rrbracket))\\
&= \llbracket\text{with}\rrbracket\left(\overset{\substack{num\\phr}}{\underset{ett}{\lambda \mathrm{P}}}\left[\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right](\llbracket\text{four fingers}\rrbracket)\right)\\
&= \llbracket\text{with}\rrbracket\left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\\
&= \text{arglift}(\llbracket\text{with}\rrbracket)\left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\\
&= \underset{\langle\vec{b},\langle et,\langle\vec{a},t\rangle\rangle\rangle}{\lambda \mathscr{R}}\left[\underset{b}{\lambda \vec{B}}\underset{ettt}{\lambda \mathscr{P}}\underset{\vec{a}}{\lambda \vec{A}}\left[\mathscr{P}\left(\underset{et}{\lambda \mathrm{X}}\left[\mathscr{R}(\vec{B})(\mathrm{X})(\vec{A})\right]\right)\right]\right](\llbracket\text{with}\rrbracket)\\
&\qquad \left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\\
&\quad \ldots \text{so } \vec{b} \text{ is null and } \vec{a} \text{ is } et \ldots\\
&= \underset{etett}{\lambda \mathscr{R}}\left[\underset{ettt}{\lambda \mathscr{P}}\underset{et}{\lambda \mathrm{A}}\left[\mathscr{P}\left(\underset{et}{\lambda \mathrm{X}}\left[\mathscr{R}(\mathrm{X})(\mathrm{A})\right]\right)\right]\right](\llbracket\text{with}\rrbracket)\left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\\
&= \underset{ettt}{\lambda \mathscr{P}}\underset{et}{\lambda \mathrm{A}}\left[\mathscr{P}\left(\underset{et}{\lambda \mathrm{X}}\left[\llbracket\text{with}\rrbracket(\mathrm{X})(\mathrm{A})\right]\right)\right]\left(\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\\
&= \underset{et}{\lambda \mathrm{A}}\left[\overset{vb}{\underset{ett}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\left(\underset{et}{\lambda \mathrm{X}}\left[\llbracket\text{with}\rrbracket(\mathrm{X})(\mathrm{A})\right]\right)\right]\\
&= \underset{et}{\lambda \mathrm{A}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \underset{et}{\lambda \mathrm{X}}\left[\llbracket\text{with}\rrbracket(\mathrm{X})(\mathrm{A})\right](\mathrm{Y})\right]\right]\\
&= \underset{et}{\lambda \mathrm{A}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{with}\rrbracket(\mathrm{Y})(\mathrm{A})\right]\right]\\
&\quad \ldots \text{applying PP}\rightarrow\text{N}/\!\!_{L}\text{N rule} \ldots
\end{aligned}
$$

$$= \underset{ett}{\overset{PP}{\lambda \mathrm{P}}}\left[\underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \mathrm{P}(\mathrm{Z})\right]\right]\right]\left(\underset{et}{\lambda \mathrm{A}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{with}\rrbracket(\mathrm{Y})(\mathrm{A})\right]\right]\right)$$
$$= \underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \underset{et}{\lambda \mathrm{A}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{with}\rrbracket(\mathrm{Y})(\mathrm{A})\right]\right](\mathrm{Z})\right]\right]$$
$$= \underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{with}\rrbracket(\mathrm{Y})(\mathrm{Z})\right]\right]\right]$$
$$= \underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \llbracket\text{with}\rrbracket(\mathrm{Y})(\mathrm{Z})\right]\right]\right] \tag{6.27}$$

⟦with four fingers⟧

$$= \underset{\langle ettt,\, ett\rangle}{\llbracket\text{with}\rrbracket}(\llbracket\text{four fingers}\rrbracket)$$
$$= \llbracket\text{with}\rrbracket(\text{numlift}(\llbracket\text{four fingers}\rrbracket))$$
$$= \llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{num\,phr}{\lambda \mathrm{P}}}\left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right](\llbracket\text{four fingers}\rrbracket)\right)$$
$$= \llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)$$

. . . applying PP→N/$_L$N rule . . .

$$= \underset{ett}{\overset{PP}{\lambda \mathrm{P}}}\left[\underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \mathrm{P}(\mathrm{Z})\right]\right]\right]\left(\llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)\right)$$

☞ $$\underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)(\mathrm{Z})\right]\right]$$

$$= \underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\underset{et}{\lambda \mathrm{W}}\left[\mathrm{W} \subseteq \llbracket\text{fingers}\rrbracket \wedge |\mathrm{W}| = 4\right](\mathrm{Y}) \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)(\mathrm{Z})\right]\right]$$
$$= \underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\underset{et}{\ell \mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \llbracket\text{with}\rrbracket\left(\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}}\left[\underset{et}{\exists \mathrm{Y}}\left[\mathrm{Y} \subseteq \llbracket\text{fingers}\rrbracket \wedge |\mathrm{Y}| = 4 \wedge \mathrm{Q}(\mathrm{Y})\right]\right]\right)(\mathrm{Z})\right]\right] \tag{6.28}$$

⟦with four fingers each⟧

$$= \llbracket\text{four fingers each}\rrbracket(\underset{\langle ettt,\, ett\rangle}{\llbracket\text{with}\rrbracket})$$
$$= \underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}}\left[\underset{b}{\lambda \vec{B}}\underset{et}{\overset{rng}{\lambda \mathrm{X}}}\underset{\vec{a}}{\lambda \vec{A}}\left[\underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right](\underset{\langle ettt,\, ett\rangle}{\llbracket\text{with}\rrbracket})$$
$$= \underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}}\left[\underset{b}{\lambda \vec{B}}\underset{et}{\overset{rng}{\lambda \mathrm{X}}}\underset{\vec{a}}{\lambda \vec{A}}\left[\underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]$$
$$(\text{arglower}(\llbracket\text{with}\rrbracket))$$
$$= \underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}}\left[\underset{b}{\lambda \vec{B}}\underset{et}{\overset{rng}{\lambda \mathrm{X}}}\underset{\vec{a}}{\lambda \vec{A}}\left[\underset{e}{\forall x} \in \mathrm{X}\left[\underset{et}{\exists \mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]$$

$$\left(\underset{\langle\vec{d},\langle ettt,\langle\vec{c},t\rangle\rangle\rangle}{\lambda\mathscr{S}}\left[\lambda\underset{\vec{d}}{\vec{D}}\lambda\underset{et}{\mathrm{Z}}\lambda\underset{\vec{c}}{\vec{C}}\left[\mathscr{S}(\vec{D})\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Z)}]\right)(\vec{C})\right]\right](\llbracket\text{with}\rrbracket)\right)$$

…with $\vec{d}$ as null, and $\vec{c}$ as et…

$$= \overset{rel}{\underset{\langle et,\langle\vec{b},\langle et,\langle\vec{a},t\rangle\rangle\rangle\rangle}{\lambda\mathscr{R}}}\left[\lambda\underset{\vec{b}}{\vec{B}}\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{\vec{a}}{\vec{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]$$

$$\left(\underset{\langle ettt,\langle et,t\rangle\rangle}{\lambda\mathscr{S}}\left[\lambda\underset{et}{\mathrm{Z}}\lambda\underset{et}{\mathrm{C}}\left[\mathscr{S}\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Z)}]\right)(\mathrm{C})\right]\right](\llbracket\text{with}\rrbracket)\right)$$

$$= \overset{rel}{\underset{\langle et,\langle\vec{b},\langle et,\langle\vec{a},t\rangle\rangle\rangle\rangle}{\lambda\mathscr{R}}}\left[\lambda\underset{\vec{b}}{\vec{B}}\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\lambda\underset{\vec{a}}{\vec{A}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]$$

$$\left(\lambda\underset{et}{\mathrm{Z}}\lambda\underset{et}{\mathrm{C}}\left[\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Z)}]\right)(\mathrm{C})\right]\right)$$

…with $\vec{b}$ and $\vec{a}$ as null…

$$= \overset{rel}{\underset{\langle et,ett\rangle}{\lambda\mathscr{R}}}\left[\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\mathscr{R}(\mathrm{Y})(\{x\})\right]\right]\right]\right]$$

$$\left(\lambda\underset{et}{\mathrm{Z}}\lambda\underset{et}{\mathrm{C}}\left[\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Z)}]\right)(\mathrm{C})\right]\right)$$

$$= \overset{rng}{\lambda\underset{et}{\mathrm{X}}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\lambda\underset{et}{\mathrm{Z}}\lambda\underset{et}{\mathrm{C}}\left[\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Z)}]\right)(\mathrm{C})\right](\mathrm{Y})(\{x\})\right]\right]\right]$$

$$= \overset{rng}{\lambda\underset{et}{\mathrm{X}}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right]$$

…applying PP→N/$_L$N rule…

$$= \overset{PP}{\lambda\underset{ett}{\mathrm{Q}}}\left[\overset{noun}{\lambda\underset{et}{\mathrm{W}}}\left[\ell\underset{et}{\mathrm{Z}}[\mathrm{Z}\subseteq\mathrm{W}\wedge\mathrm{Q(Z)}]\right]\right]$$

$$\left(\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right]\right)$$

$$= \overset{noun}{\lambda\underset{et}{\mathrm{W}}}\left[\ell\underset{et}{\mathrm{Z}}\left[\mathrm{Z}\subseteq\mathrm{W}\right.\right.$$
$$\left.\left.\wedge\overset{rng}{\lambda\underset{et}{\mathrm{X}}}\left[\forall\underset{e}{x}\in\mathrm{X}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right](\mathrm{Z})\right]\right]$$

☞ $$\overset{noun}{\lambda\underset{et}{\mathrm{W}}}\left[\ell\underset{et}{\mathrm{Z}}\left[\mathrm{Z}\subseteq\mathrm{W}\wedge\forall\underset{e}{x}\in\mathrm{Z}\left[\exists\underset{et}{\mathrm{Y}}\left[\llbracket\text{four fingers}\rrbracket(\mathrm{Y})\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right]\right]$$

$$= \overset{noun}{\lambda\underset{et}{\mathrm{W}}}\left[\ell\underset{et}{\mathrm{Z}}\left[\mathrm{Z}\subseteq\mathrm{W}\right.\right.$$
$$\left.\left.\wedge\forall\underset{e}{x}\in\mathrm{Z}\left[\exists\underset{et}{\mathrm{Y}}\left[\lambda\underset{et}{\mathrm{X}}[\mathrm{X}\subseteq\llbracket\text{fingers}\rrbracket\wedge|\mathrm{X}|=4](\mathrm{Y})\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right]\right]$$

$$= \overset{noun}{\lambda\underset{et}{\mathrm{W}}}\left[\ell\underset{et}{\mathrm{Z}}\left[\mathrm{Z}\subseteq\mathrm{W}\right.\right.$$
$$\left.\left.\wedge\forall\underset{e}{x}\in\mathrm{Z}\left[\exists\underset{et}{\mathrm{Y}}\left[\mathrm{Y}\subseteq\llbracket\text{fingers}\rrbracket\wedge|\mathrm{Y}|=4\wedge\llbracket\text{with}\rrbracket\left(\lambda\underset{ett}{\mathrm{P}}[\mathrm{P(Y)}]\right)(\{x\})\right]\right]\right]\right] \tag{6.30}$$

$$
\begin{aligned}
&\llbracket \text{Every kid bought three books each.} \rrbracket \\
&\quad = \quad \llbracket \text{every kid} \rrbracket (\llbracket \text{bought three book each} \rrbracket) \\
&\quad = \quad \lambda \underset{ett}{\overset{vb}{Q}} \left[\forall \underset{et}{z} \in \llbracket \text{kid} \rrbracket \left[Q(\{z\}) \right] \right] \\
&\qquad\qquad \left(\lambda \underset{et}{\overset{rng}{X}} \left[\underbrace{|X| > 1}_{\substack{plurality \\ stipulation}} \wedge \forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \right) \\
&\quad = \quad \forall \underset{et}{z} \in \llbracket \text{kid} \rrbracket \left[\lambda \underset{et}{\overset{rng}{X}} \left[|X| > 1 \wedge \forall \underset{e}{x} \in X \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] (\{z\}) \right] \\
&\quad = \quad \forall \underset{et}{z} \in \llbracket \text{kid} \rrbracket \left[|\{z\}| > 1 \wedge \forall \underset{e}{x} \in \{z\} \left[\exists \underset{et}{Y} \left[\llbracket \text{three books} \rrbracket (Y) \wedge \llbracket \text{bought} \rrbracket (Y)(\{x\}) \right] \right] \right] \qquad (6.31)
\end{aligned}
$$

Appendix C

Definitions

For convenient reference, we collect here several of the definitions given throughout the thesis.

Chapter 4

$$[\![\text{each}]\!]_{\text{Zimm}} \quad = \quad \lambda P.\forall z[(z \in Z_i) \rightarrow \exists x[P(x) \wedge R_j(z,x)]] \tag{4.1}$$

Chapter 5

$$\underset{etett}{[\![N]\!]} \quad = \quad \underset{et}{\overset{noun}{\lambda P}} \left[\underset{et}{\lambda \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge |\mathrm{X}| = N \right] \right] \tag{5.7}$$

$$\underset{\langle ett,\, ettt\rangle}{\text{numlift}} \quad = \quad \underset{ett}{\overset{\substack{num\\phr}}{\lambda \mathrm{P}}} \left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\underset{et}{\exists \mathrm{Y}} \left[\mathrm{P(Y)} \wedge \mathrm{Q(Y)} \right] \right] \right] \tag{5.10}$$

$$\underset{\langle et,\, \langle ett,\, t\rangle\rangle}{[\![\text{some}]\!]} \quad = \quad \underset{et}{\overset{noun}{\lambda P}} \left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\underset{et}{\exists \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge \mathrm{Q(X)} \right] \right] \right] \tag{5.16}$$

$$\underset{\langle et,\, \langle ett,\, t\rangle\rangle}{[\![\text{no}]\!]} \quad = \quad \underset{et}{\overset{noun}{\lambda P}} \left[\underset{ett}{\overset{vb}{\lambda \mathrm{Q}}} \left[\neg \underset{et}{\exists \mathrm{X}} \left[\mathrm{X} \subseteq P \wedge \mathrm{Q(X)} \right] \right] \right] \tag{5.17}$$

$$\text{arglift} \quad = \underset{\langle \vec{b},\, \langle et,\, \langle \vec{a},\, t\rangle\rangle\rangle}{\lambda \mathscr{R}} \left[\underset{\vec{b}}{\lambda \vec{B}} \underset{ettt}{\lambda \mathscr{P}} \underset{\vec{a}}{\lambda \vec{A}} \left[\mathscr{P} \left(\underset{et}{\lambda \mathrm{Y}} \left[\mathscr{R}(\vec{B})(\mathrm{Y})(\vec{A}) \right] \right) \right] \right] \tag{5.21}$$

$$\text{objlift} \quad = \quad \underset{\langle et,\, \langle et,\, t\rangle\rangle}{\overset{tr.\ verb}{\lambda \mathscr{R}}} \left[\underset{ettt}{\overset{obj}{\lambda \mathscr{P}}} \left[\underset{et}{\overset{sbj}{\lambda \mathrm{X}}} \left[\mathscr{P} \left(\underset{et}{\overset{obj}{\lambda \mathrm{Y}}} \left[\mathscr{R}(\mathrm{Y})(\mathrm{X}) \right] \right) \right] \right] \right] \tag{5.22}$$

Chapter 6

$$[\![\text{each}]\!]_{\text{simple}} = \underset{ett}{\overset{dist}{\lambda \mathrm{P}}}\left[\underset{etett}{\overset{rel}{\lambda \mathscr{R}}}\left[\underset{et}{\overset{rng}{\lambda \mathrm{X}}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\{x\})\right]\right]\right]\right]\right] \quad (6.2)$$

$$[\![\text{each}]\!]_{\text{full}} = \underset{ett}{\overset{dist}{\lambda \mathrm{P}}}\left[\underset{\langle et, \langle \vec{b}, \langle et, \langle \vec{a}, t\rangle\rangle\rangle\rangle}{\overset{rel}{\lambda \mathscr{R}}}\left[\lambda \underset{\vec{b}}{\vec{B}} \underset{et}{\overset{rng}{\lambda \mathrm{X}}} \lambda \underset{\vec{a}}{\vec{A}}\left[\forall \underset{e}{x} \in \mathrm{X}\left[\exists \underset{et}{\mathrm{Y}}\left[\mathrm{P}(\mathrm{Y}) \wedge \mathscr{R}(\mathrm{Y})(\vec{B})(\{x\})(\vec{A})\right]\right]\right]\right]\right] \quad (6.8)$$

$$\mathrm{PP} \rightarrow \mathrm{N}/_{L}\mathrm{N} = \underset{ett}{\overset{PP}{\lambda \mathrm{P}}}\left[\underset{et}{\overset{noun}{\lambda \mathrm{X}}}\left[\ell \underset{et}{\mathrm{Z}}\left[\mathrm{Z} \subseteq \mathrm{X} \wedge \mathrm{P}(\mathrm{Z})\right]\right]\right] \quad (6.26)$$

$$\text{arglower} = \underset{\langle \vec{b}, \langle ettt, \langle \vec{a}, t\rangle\rangle\rangle}{\lambda \mathscr{R}}\left[\lambda \underset{\vec{b}}{\vec{B}} \lambda \underset{et}{\mathrm{X}} \lambda \underset{\vec{a}}{\vec{A}}\left[\mathscr{R}(\vec{B})\left(\lambda \underset{ett}{\mathrm{P}}\left[\mathrm{P}(\mathrm{X})\right]\right)(\vec{A})\right]\right] \quad (6.29)$$

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