The Principle of Indifference

The Principle of Indifference (or Insufficient Reason) is often stated as something like:

> Absent evidence to the contrary, all outcomes of a trial should be assumed to have equal probability.

It is virtually always attributed to Simon Pierre, Marquis de Laplace, though rarely with an actual cite; a few give his *Essai philosophique sur les probabilités* (1814) as the source. While this document remains a classic in the field (and an excellent read), the principle is not entirely original to this work, not named in this work, and not even really stated as a principle in itself, though it is noted in the discussion.

The first to enunciate the principle in some form was probably Jacques[1] Bernoulli, in his *Ars conjectandi* (1713) (emphasis mine):

> Similarly, the number of possible cases is known in drawing a white or a black ball from an urn, and one can assert that any ball is equally likely to be drawn; for it is known how many balls of each kind are in the jar, *and there is no reason why this or that ball should be drawn more readily than any other.*[2]

The connection between this citation and the Principle has been made in a number of places, but while the quote is definitely getting at the idea, it is made only in terms of the specific case under consideration. His nephew Daniel Bernoulli stated the principle in a more general form, and more clearly recognisable as a statement of the Principle in *Specimen theoriae novae de mensura sortis* (1738):

> Since there is no reason to assume that of two persons encountering identical risks, either should expect to have his desires more

[1] a.k.a. James, Jakob, or Jacobi

[2] English translation found in (Calinger, 1995), created from the German translation by R. Haussner. In the original Latin: "Sic itidem noti sunt numeri casuum ad educendam ex urna schedulam albam nigramve, & notum est omnes æquè possibiles esse; quia nimirum determinati notique sunt numeri schedarum utriusque generis, *nullaque perspicitur traio, cur hæc vel illa potius exire debeat quàm quælibet alia.*"

> closely fulfilled, the risks anticipated by each must be deemed equal in value.[3]

By comparison, Laplace (1814) leaves it mostly implicit in his First Principle of the Calculus of Probabilities: "The very definition of probability. . . is the ratio of the number of favourable cases to that of all possible cases."[4] Earlier he gives most of it in the midst of some discussion (emphasis mine):

> The theory of chances consists of reducing all events of the same kind to a certain number of equally possible cases, that is, cases *about whose existence we are equally uncertain*; and of determining the number of cases favourable to the event whose probability is sought. The ratio of this number to that of all possible cases is the measure of this probability, which is thus only a fraction whose numerator is the number of favourable cases, and whose denominator is the number of all possible cases.[5]

This is somewhat more clearly related to the Principle as we know it. Unquestionably, Laplace understood and believed in the underlying fact of the principle (as early as 1776: "if we see no reason why one case should happen more than the other" (Hacking, 1975, p. 131)). However, he never really states it as a principle in its own right, acknowledging that the important notion is not just equiprobability, but the indifference leading to the (presumed) equiprobability.

The first real statement of the Principle as such seems to come from Johannes von Kries. In his *Die Principien der Wahrscheinlichkeits-Rechnung* (1886), he states in Chapter I §4 (emphasis his):

> When now the logical [consequence] of our knowledge should present itself in the performance of a number of equally possible cases, thus arises without difficulty the explanation, *that two or more cases are to be regarded as equally possible, when in their respective circumstances we can find no reason to maintain one as possibly more probable than some other.*[6]

In the paragraph after this clear statement of the Principle, he names it (again, emphasis his):

[3]Translation by (Sommer, 1954). Original not available.

[4]Translations from this work are based on (Dale, 1995). The original: "La définition même de la probabilité...est le rapport du nombre des cas favorables, à celui de tous les cas possibles."

[5]In the original French: "La théorie des hasards consiste à reduire tous les évènemens du même genre, à un certain nombre de cas également possibles, c'est-à-dire, tels que nous soyons également indécis sur leur existence; et à déterminer le nombre de cas favorable à l'évènement dont on cherche la probabilité. Le rapport de ce nombre à celui de tous les cas possibles, est la mesure de cette probabilité qui n'est ainsi qu'une fraction dont le numérateur est le nombre des cas favorables, et dont le dénominateur est le nombre de tous les cas possibles."

[6]All translations from this work are my own. In the original German: "Wenn nun das logische Verhalten unseres Wissens in der Aufführung einer Anzahl von gleich möglichen Fällen sich darstellen soll, so ergiebt sich ohne Schwierigkeit die Erklärung, *dass als gleich möglich zwei oder mehrere Fälle anzusehen sind, wenn in dem jeweiligen Stande unserer Kenntnisse sich kein Grund findet, unter ihnen einen für wahrscheinlicher als irgend einen anderen zu halten.*"

> We want to briefly designate... that principle, on which the calculation of probability is based, as the *Principle of Insufficient Reason*.[7]

Obviously, von Kries was aware of Laplace's work, and knew that the underlying implications were not original; he states as much when relating the history of probability theory in Chapter X, at the end of §3:

> With that, we reach essentially the point of view on which Laplace stands. In his writings, we find the short explanation: "equally possible cases, that is, cases about whose existence we are equally uncertain", a view in accordance with the Principle of Insufficient Reason that we have mentioned.[8]

Finally, early in the twentieth century, John Maynard Keynes gives the principle its now-more-familiar name in his *Treatise on Probability* (1921):

> The Principle of Indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability. Thus *equal* probabilities must be assigned to each of several arguments, if there is an absence of positive ground for assigning *unequal* ones.

[7]The original German: "Wir wollen... das Princip, auf welches sie die Wahrscheinlichkeits-Rechnung basirt, als *Princip des mangelnden Grundes* [bezeichnen]."

[8]The original German: "Hiermit ist im Wesentlichen der Standpunkt erreicht, auf welchem auch Laplace steht. Bei diesem finden wir die kurze Erklärung: "cas également possibles, c'est à dire tels que nous soyons également indécis sur leur existence," eine Auffassung, welche mit dem von uns so genannten Princip des mangelnden Grundes zusammentrifft."

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