Homework 3

Due: 19th of April, 2018

This homework may be done in groups. Everyone in the group is responsible for knowing how to solve all of the problems. Your group needs to meet (as a group!) at least twice.

Problems

1. Let G be a polyhedron, all of whose faces are bounded by pentagons and hexagons.
   (a) Show that G must have at least 12 pentagonal faces
   (b) If, in addition, there are exactly three faces meeting at each vertex, prove that G has exactly 12 pentagonal faces.
2. Let G be a planar graph containing no triangles.
   (a) Use Euler's Formula to show that G contains a vertex whose degree is at most 3.
   (b) Use induction to deduce that G is 4-colorable.
3. Let G be a graph with at least 11 vertices and $\bar{G}$ denote its complement
   (a) Prove that G and $\bar{G}$ cannot both be planar.
   (b) Find a graph G with eight vertices such that G and $\bar{G}$ are both planar
4. The *girth* of a graph is the length of its shortest cycle.
   (a) If G is a connected plane graph of girth 5, then show that $m \leq \frac{5(n-2)}{3}$
   (b) Show that the Petersen graph is non-planar
5. Show that the Grinberg graph is not hamiltonian.