

Homework 2

Due: 27th of March, 2018

This homework may be done in groups. Everyone in the group is responsible for knowing how to solve all of the problems. Your group needs to meet (as a group!) at least twice.

Problems

1. Prove that the decision version of the Traveling Salesman Problem is NP-Complete
2. (a) Prove that if G is a bipartite graph with an odd number of vertices, then G is non-hamiltonian.
(b) Show that if n is odd, then it is not possible for the knight to visit all the squares of an $n \times n$ chess board exactly once and return to his starting position.
3. (a) Is it possible for a knight to visit all the squares of an 8×8 chess board exactly once and return to his starting position?
(b) Is it possible for a knight to travel around a regular chess board in such a way that every possible move occurs exactly once?
4. (a) Let G be a graph with $n \geq 3$ vertices and $(n - 1)(n - 2)/2 + 2$ edges. Show that G is hamiltonian.
(b) Give an example of a non-hamiltonian graph with n vertices and $(n - 1)(n - 2)/2 + 1$ edges.
5. (a) Show that the line graph of an eulerian graph is eulerian.
(b) If the line graph of a graph G is eulerian, then must G be eulerian? Why or why not?
6. There are 27 one inch cubes of differently flavored cheese stacked in a $3 \times 3 \times 3$ cube. A mouse, starting at a corner, samples each type of cheese once and only once, but he may go only from one type of cheese to another type of cheese if the cheese share a face. Can this mouse end with the cheese in the middle of the cube? Why or why not?