

Homework 1

6th of February, 2018

This homework is to be done in the groups of up to three. Everyone in the group is responsible for knowing how to solve all of the problems. Your group needs to meet (as a group!) at least twice. At least two members should be present to get help on a homework problem.

Problems

1. Let \mathbf{G} be a graph. Show that \mathbf{G} and $\overline{\mathbf{G}}$ cannot both be disconnected.
2. Let \mathbf{G} be a graph with two or more vertices. Prove that \mathbf{G} must contain two or more vertices of the same degree.
3. Let \mathbf{G} be a graph with n vertices and m edges and with an adjacency matrix $A = [a_{ij}]$. Let $A^2 = [b_{ij}]$ and $A^3 = [c_{ij}]$. ($\text{Tr}()$ is the trace of the matrix – the sum of the diagonal)
 - (a) Show that $b_{ii} = \deg(v_i)$ and that $\text{Tr}(A^2) = 2m$
 - (b) Show that the number of triangles in \mathbf{G} containing v_i , is $\frac{1}{2}c_{ii}$ and that the total number of triangles in \mathbf{G} is $\frac{1}{6}\text{Tr}(A^3)$.
4. Show that if \mathbf{G} were self-complementary with n vertices, then either $n = 4k$ or $n = 4k + 1$.
5. Draw all trees of size 6, 7, and 8.
6. Find and take a picture (with all group members) of four trees that are not trees. Your trees should be on the Longwood campus.