Project Steinhaus

11th of April, 2016

Your second project is to investigate the class of Steinhaus Graphs.

Steinhaus Graphs

Steinhaus graphs are a class of graphs where the adjacency matrix is constructed from a generator. Let $T = a_1 ... a_{n-1}$ be an $n-1$ string of binary values. A Steinhaus graph, $H_{n,T}$, is the graph on n vertices with the generator T. We often write T as the decimal representation of the binary string. The Steinhaus graph's adjacency matrix is defined as:

$$a_{i,j} = \begin{cases} 0 & \text{if } 0 \le i = j < n; \\ (a_{i-1,j-1} + a_{i-1,j}) \text{mod} 2 & \text{if } 0 < i < j < n; \\ a_{j,i} & \text{if } 0 \le j < i < n; \end{cases} \tag{1}$$

An Example

The Steinhaus graph with eight vertices and generator 22 has the following adjacency matrix.

$$H_{8,22} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The Project

Characterize

For each of the Steinhaus graphs with eight vertices you will identify the following traits and give a reason:

1. Hamiltonian
2. Eulerian
3. Regular
4. Bipartite
5. Connected
6. Isomorphic (give the generators of the isomorphic graphs)
7. Tree

Prove

In addition you will prove the following statements about Steinhaus graphs.

1. The number of doubly symmetric Steinhaus graphs is $2^{\lfloor n/2 \rfloor}$
2. If G is a Steinhaus graph, then G-0 and G-(n-1) are Steinhaus graphs.
3. A Steinhaus graph and its partner are isomorphic.
4. Every Steinhaus graph is connected except for $H_{n,0}$