

# Homework 3

*4th of April, 2016*

This homework may be done in groups. Everyone in the group is responsible for knowing how to solve all of the problems. Your group needs to meet (as a group!) at least twice.

## Problems

1. Prove that the Undirected Hamiltonian Cycle problem is NP-Complete
2. Prove that the decision version of the Traveling Salesman Problem is NP-Complete
3. (a) Prove that if  $G$  is a bipartite graph with an odd number of vertices, then  $G$  is non-hamiltonian.  
(b) Show that if  $n$  is odd, then it is not possible for the knight to visit all the squares of an  $n \times n$  chess board exactly once and return to his starting position.
4. (a) Is it possible for a knight to visit all the squares of an  $8 \times 8$  chess board exactly once and return to his starting position?  
(b) Is it possible for a knight to travel around a regular chess board in such a way that every possible move occurs exactly once?
5. (a) Let  $G$  be a graph with  $n \geq 3$  vertices and  $(n - 1)(n - 2)/2 + 2$  edges. Show that  $G$  is hamiltonian.  
(b) Give an example of a non-hamiltonian graph with  $n$  vertices and  $(n - 1)(n - 2)/2 + 1$  edges.
6. (a) Show that the line graph of an eulerian graph is eulerian.  
(b) If the line graph of a graph  $G$  is eulerian, then must  $G$  be eulerian? Why or why not?
7. There are 27 one inch cubes of differently flavored cheese stacked in a  $3 \times 3 \times 3$  cube. A mouse, starting at a corner, samples each type of cheese once and only once, but he may go only from one type of cheese to another type of cheese if the cheese share a face. Can this mouse end with the cheese in the middle of the cube? Why or why not?