Homework 2

18th of March, 2016

This homework may be done in groups. Everyone in the group is responsible for knowing how to solve all of the problems. Your group needs to meet (as a group!) at least twice.

Problems

1. Find an efficient algorithm to compute κ (the vertex connectivity), λ (the edge connectivity), or show that computing these parameters lies in the murky realm of NP-Hard.
2. Show that $\kappa \leq \lambda$.
3. Let G be a graph and let S be a set of $\lambda(G)$ edges whose removal from G disconnects G. Show that $G - S$ has exactly two components.
4. Find a graph with 10 vertices that realizes $(2, 3, 4, 6)$.
5. There are generalizations of complete graphs and one of these are the Harary graphs, $H_{n,k}$ where n is the number of vertices in the graph and k is less than n. For k even, $H_{n,k}$ has vertices $\{0, 1, \ldots, n-1\}$ and edge-set

$$\Big\{\{i, (i+j) \bmod n\} : 0 \leq i < n; 0 < j \leq k/2\Big\}$$

and if n is even and k is odd, then $H_{n,k}$ is $H_{n,k-1}$ with the additional edges

$$\Big\{\{i, (i+n/2)\} : 0 \leq i < n/2\Big\}.$$

If nk is odd, then we add the following edges to $H_{n,k-1}$

$$\Big\{\{i, (i+(n-1)/2)\} : 0 \leq i < n/2\Big\}.$$

   (a) Draw $H_{6,i}$ for $0 \leq i < 6$.
   (b) Draw $H_{7,i}$ for $0 \leq i < 7$.
   (c) Show that for $k > 1$ $\kappa(H_{n,k}) = \lambda(H_{n,k}) = \delta(H_{n,k}) = k$ and if nk is even, then $\Delta(H_{n,k}) = k$ and if nk is odd, then $\Delta(H_{n,k}) = k + 1$.

6. Find a graph with 100 vertices that realizes $(2, 3, 4, 6)$. Hint: Connect K_5 and $H_{95,4}$.
7. Are the following 4-tuples realizable? If so, then what is the smallest possible number of vertices needed to realize the 4-tuple and show that no smaller number of vertices will work. If not, then show why not.

   (a) $(4, 6, 9, 9)$
   (b) $(4, 5, 8, 8)$
   (c) $(4, 5, 9, 9)$