Homework 1

Due: 12 Sep 2017

Problem 1.1

Consider an image with eight colours, with the following counts for each colour:

transparent	12,000
opaque white	15,000
semitransparent white	6,000
light green	12,600
dark green	8,400
cyan	10,000
yellow	16,000
black	20,000

a. Build a Huffman tree for these frequency counts. Draw it.

b. The first non-metadata line of the file contains, in sequence, 2 transparent pixels, one semitransparent white pixel, two opaque white pixels, and five black pixels. Encode this sequence into the Huffman code implied by the tree you drew. Though this will technically just be ones and zeroes, indicate which groups of bits correspond to each pixel.

Problem 1.2

We run the Huffman algorithm on a set of symbols paired with their relative frequencies, to produce a trie that creates a correspondence between the symbols and variable-length binary strings. Consider now what would be required to actually implement the Huffman algorithm: identify (in the language of your choice or pseudocode) what class(es) or struct(s) you would need to create to represent the tree and support the algorithm (including their fields); and what library ADTs would you make use of, and how.

(Note: this is essentially the early design phase of a Huffman implementation, but that's all—no code is expected here!)

GENERAL NOTE ABOUT PROBABILITY PROBLEMS:

- Include work in symbolic form (e.g. $p(X = \text{foo}|Y = 3)$). I should be able to tell where every number came from. You can abbreviate (e.g. $p(\text{foo}|3)$) as long as it's clear what is meant.
- Sanity-check your answers. If your answer seems crazy but you can't find the error, at least make it clear you know there's a problem.

Problem 1.3

A standard deck of playing cards has 52 cards in four suits, each suit with cards numbered 2–10 and "face cards" labelled jack, queen, king, and ace. A "pinochle deck" has 48 cards: each of the four suits has only 9 and 10 plus the face cards, and each card appears twice in the deck.[1]

Assuming in each case that the relevant deck is well-shuffled, identify the following probabilities. Show your work by making it clear where each number in your probability comes from.

a. p(R=ace | D=standard): In a standard deck, the probability of drawing any card whose rank is 'ace'

b. p(R=ace | D=pinochle): In a pinochle deck, the probability of drawing any card whose rank is 'ace'

c. p(F=true, C=red | D=pinochle): the probability of drawing a card whose rank is one of the face cards and whose suit is one of the red suits, from a pinochle deck

d. p(R=A, S=H | F=true): the probability of drawing the ace of hearts given that the card drawn is a face card

e. If there are 3 standard decks and 1 pinochle deck in a pile, and you pick one at random before drawing a card from that deck, what is p(F=true), i.e. the overall probability of drawing a face card?

Collaboration policy: group work! If you work with other people on this homework, hand in one copy and put all your names on top. There will be a revision cycle for this.

[1] I swear I am not making this up. It's pronounced "PEE-nuckle".